FBD - The Free Body Diagram Method. Kinematic and Dynamic Modeling of a Six Leg Robot

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Abstract

The Free Body Diagram method, based on the dynamic equations of isolated rigid bodies, is used to overcome the difficulties in dynamic modeling of legged robots. This article presents a simulator for a six leg machine. Both kinematic and dynamic models are developed. Kinematic equations are derived with Denavit-Hartenberg method. The Free Body Diagram method is used to obtain the dynamic model. Some results of simulation are presented.

1 Introduction

Most of the vehicles that we are familiar with use wheels for their locomotion. Wheeled vehicles can achieve high speed motion with a relative low control complexity. Unfortunately they present several limitations in rough and irregular surfaces. Even with complex suspension systems they are only able to overcome relatively small irregularities on the terrain. The US army estimates that the wheel can only reach 50% of the places on earth. Whenever environment is a concern, the destruction made in building suitable tracks is another problem ([6][7][4]). The legged locomotion is one alternative that overcomes these difficulties. It introduces more flexibility and soil adaptation at the cost of lower speed and increased control complexity. Legged vehicles can walk on rough and irregular surfaces with a minimum of destruction and a high degree of softness. This explains the importance of legged robots on mobile robotics research.

The legged locomotion on natural terrain presents a set of complex problems (foot placement, obstacle avoidance, load distribution by the supports, general vehicle stability, etc) that must be taken into account both in mechanical construction of vehicles and in development of control strategies. One way to handle these issues is using models that mathematically describe the different situations. Therefore modelization becomes a useful tool in understanding systems complexity and in testing and simulating different control approaches.

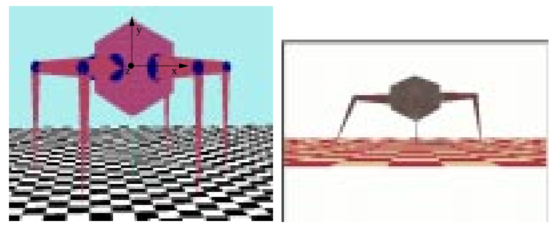

Figure 1: The hexapodal 3D structure. Simplified 2D structure

Modelization techniques for mechanical structures are developed in this paper. The Denavit-Hartenberg method is used in deriving a 3D kinematic model of a six leg robot. Dynamic modelization is performed using the Free Body Diagram method (FBD). The FBD method is introduced as an alternative to Lagrangian Formalism and is based in the dynamics of isolated rigid bodies. A simulator is built to validate the achieved models. Some simulation results are presented in section 6.

2 The mechanical structures

The considered 3D structure is formed by a central body, with an hexagonal shape and six legs. The legs are similar and simetrically distributed around the body (Fig.1). Each leg is composed by two links and three rotary joints. Two of these joints are located at the junction of the leg with central body (horizontal

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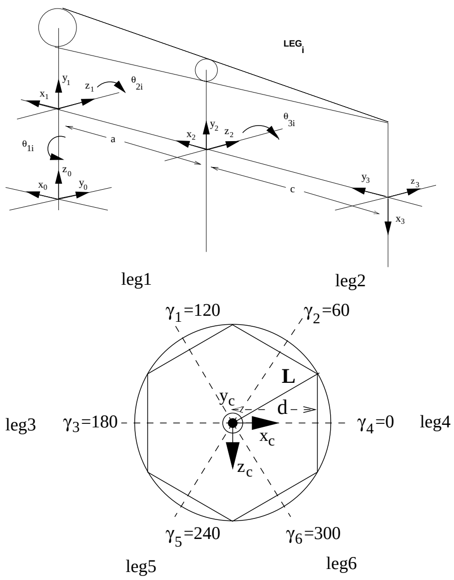


Figure 2: The coordinate systems defined for each leg. Legs' locations around the central body.

(θ_{1i}) and vertical (θ_{2i}) rotation). The third joint is located at the knee, connecting the upper and lower link (vertical rotation (θ_{3i})). Therefore each leg has 3 DOF (degree of freedom).Considering six legs and the additional 6 DOF for central body translation and rotation, the system has a total of 24 DOF.

The dynamic modeling of the 3D structure with six legs is a huge problem that would lead to a great amount of equations. Thus, to explain dynamic modeling using FBD approach, a simplified planar structure is considered. However, the formalism of FBD method can be extended to 3D structures. The simplified 2D structure has two legs and a central body. Each leg is composed by two links and two rotary joints (θ_{1i} disappears). Considering central body with 3 DOF (translation in X and Y and rotation around Z), the system has a total of 7 DOF.

3 The kinematic equations

3.1 Direct kinematics of 3D structure

$$
{}^{\mathbf{0}}\boldsymbol{A_1} = \begin{bmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) & 0 \\ \sin(\theta_1) & 0 & -\cos(\theta_1) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}
$$

$$
{}^{\mathbf{1}}\boldsymbol{A_2} = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & -a.\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & -a.\sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}
$$

$$
{}^{\mathbf{2}}\boldsymbol{A_3} = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & c.\cos(\theta_3) \\ \sin(\theta_3) & \cos(\theta_3) & 0 & c.\sin(\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}
$$

$$
\vec{\boldsymbol{r_0}} = \boldsymbol{F}(\theta_1, \theta_2, \theta_3) = {}^{\mathbf{0}}\boldsymbol{A_1}(\theta_1).{}^{\mathbf{1}}\boldsymbol{A_2}(\theta_2).{}^{\mathbf{2}}\boldsymbol{A_3}(\theta_3).[0,0,0,1]^t \tag{4}
$$

The Denavit-Hartenberg (D-H) method is one of the most popular technics used in kinematic modeling of manipulators [1][3][5]. The described robot legs are similar to simple manipulators with 3 DOF. Therefore D-H method can be used to compute the transformation matrices between referential frames (Fig.2). Derived transformation matrices are presented in equations 1, 2, 3, where a and c are the length of links. Leg direct kinematic problem can be solved by these matrices. Function (4) shows it by computing tip coordinates on the base system given an arbitrary triad of joint angles $(\theta_1, \theta_2, \theta_3)$.

$$
{}^{\boldsymbol{c}}\boldsymbol{A_{0i}} = \begin{bmatrix} -\cos(\gamma_i) & \sin(\gamma_i) & 0 & d.\cos(\gamma_i) \\ 0 & 0 & 1 & 0 \\ \sin(\gamma_i) & \cos(\gamma_i) & 0 & -d.\sin(\gamma_i) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}
$$

The six legs and the central body must be integrated to solve the global kinematic problem. Consider the referential located at the center of the body (Fig.2). Leg_i coordinates in body referential are obtained using the transformation matrix of equation 5. Note that $d = L.cos(\frac{\pi}{6})$ and each leg has a different γ_i associated with it.

For simulation purposes it is important to be able to compute robot coordinates in an inertial frame located somewhere in space. The transformation matrix ${}^{\mathbf{i}}\mathbf{A_c}$ between body referential and the inertial frame depends of the 6 DOF of the robot central body. Three DOF are the angular positions (ϕ_x, ϕ_y, ϕ_z) of the body around the inertial axes. The other three are the coordinates of the mass center (R_x, R_y, R_z) in inertial frame. The considered rotation sequence is (Y,Z,X) (see [1] for more details).

3.2 Direct kinematics of 2D structure

Consider the planar structure composed by a central body and a pair of legs (Fig.3). The legs used in this example are the 3 and 4 of the equivalent 3D structure (Fig.2). It is necessary to reach a kinematic

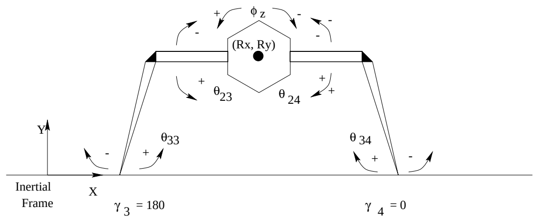


Figure 3: Independent variables of 2D kinematic model.

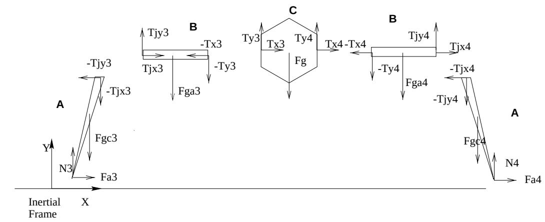


Figure 4: Free Body Diagram

model of the system and to select the independent or generalized kinematic variables.

Planar transformation matrices $^{\mathbf{2}}\mathbf{A_3}$, $^{\mathbf{1}}\mathbf{A_2}$, $^{\mathbf{c}}\mathbf{A_1}$, $^{\mathbf{i}}\mathbf{A_c}$ are derived from matrices 3D transformation matrices. Note that if values of - R_x, R_y, ϕ_z, θ_{23}, θ_{24}, θ_{33}, θ_{34} - are known at a given instant of time, structure position can be determined in an unambiguous way. These variables are the seven independent kinematic variables (or the independent generalized variables of Lagrange Formalism [11]).

4 The dynamic equations

Dynamic modeling of mechanical structures can be a complex problem. In robotics there are two main classical methodologies used for dynamic modeling: Lagrange and Newton-Euler.

The Lagrange approach is based on the energy principles. It works with scalar quantities, instead of vectors, handling the internal forces between the elements of the system in an implicit way. This method, although computationally expensive, can be particularly useful when a state space model is intended.[1] [10] [11].

The Newton-Euler method applies the vectorial dynamic equations to each element of the structure. The final system is achieved by joining all the elements equations. The internal forces are handled in an explicit way, as well as inertial and Coriolis forces. Most of the times Newton-Euler technique is difficult to use in modeling interacting structures like legged robots.[1] [3]

In this work an alternative method, called Free Body Diagram (FBD), is used to model legged robots. It is based in dynamic equations of isolated rigid bodies (a standard in mechanics) and integrates some concepts of the last two methods. Considering the 3D six leg structure, dynamic model derivation would inevitably lead to a great number of equations and variables. Thus, and without loss of generality, FBD approach is explained considering a planar structure with two legs. All the enunciated formalism can be extended to a 3D structure.

5 Introduction to the FBD method

$$\sum F_x^{ext} = M.a_x \tag{6}$$

$$\sum F_y^{ext} = M.a_y \tag{7}$$

$$\vec{\boldsymbol{M}}_{\boldsymbol{z}} = \sum_i \vec{\boldsymbol{r}}_{\boldsymbol{i}} \times \vec{\boldsymbol{F}}_{\boldsymbol{i}}^{\boldsymbol{ext}} \tag{8}$$

$$M_z = I_{CM}.\gamma \tag{9}$$

The FBD method is based in rigid body dynamics. Given a 2D body, its position in an inertial frame is determined in an unique way by the XY coordinates of the center of mass (CM) and a rotation angle. Equations 6, 7, 8 and 9 describe the dynamic behavior of the body when a set of external forces F_i^{ext} is applied. Equations 6 and 7 calculate the translational motion of the CM due to the applied resultant force (in X and Y directions). Equation 8 computes the force moment M_z and equation 9 determines the angular acceleration. Notice that M is the mass and I_{CM} is the inertial moment for rotation around an axis parallel to inertial frame Z axis and passing through CM. If an inertial moment I_P is considered the rotation axis will pass through a given point P instead of CM.

This formulation can be extended to a 3D body. Its position is determined by XYZ coordinates of CM, thus it has three dynamic equations for translation. If the body motion is completely free and can rotate around inertial X, Y and Z directions, three rotation equations are needed. For the general case the dynamic behavior is described by six equations. Considering the example, 2D structure is formed by five rigid bodies: the central body (cp), the superior links (a3, a4) of both legs and the inferior links (c3, c4). Each element must be isolated to build the FBD

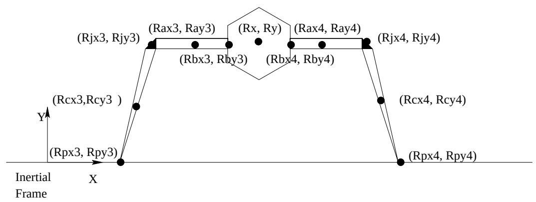


Figure 5: Auxiliary points

diagram with all the applied forces (Fig.4). The external forces to the structure are the gravitic forces and the friction forces on the tips (friction on the joints is not considered). The contact forces between the different elements are internal to the system.

5.1 Auxiliary points coordinates

To avoid dealing with inertial forces and complex transformations, dynamic equations are derived in an inertial referential located outside the structure. Therefore the inertial coordinates of the points where forces are applied must be calculated . These points are: the mass centers (gravitical forces), the contact points of the links(tension forces) and the supports (friction forces)(Fig. 5). They are essential to compute the rotation force moments . The arm vectors coordinates (r_i in equation 8) can be determined by subtracting pairs of points coordinates in the inertial frame referential.

With 2D transformation matrices the derivation of inertial points coordinates as a function of the seven independent kinematic variables is straight forward. These functions describe the restrictions in motion imposed by structure configuration. In the example of 2D structure the five rigid bodies are connected and, consequently, the motion of each one is dependent of the others.

5.2 Rotation versors of isolated rigid bodies

$$\vec{M}^r = \vec{M}.\vec{vers} \tag{10}$$

For each isolated body, the resultant force moment ($\vec{\mathbf{M}}(\mathbf{M_x}, \mathbf{M_y}, \mathbf{M_z})$) is calculated in an inertial frame placed outside the structure. To determine the angular acceleration it is necessary to compute the component of force moment ($\vec{\mathbf{M}}^{\mathbf{r}}(\mathbf{M^r_x}, \mathbf{M^r_y}, \mathbf{M^r_z})$) with the direction of body rotation axis (equation 9). In D-H method the Z axis of the referential frames attached to the each joint is coincident with the joint rotation axis (Fig 2). Thus, rotation axis versor $\mathbf{\vec{vers}}$ is determined by computing Z versor coordinates in the inertial referential frame. These are easily derived using the kinematic transformation matrices. $\vec{\mathbf{M}}^{\mathbf{r}}$ is calculated in equation 10 using scalar product.

Considering the 2D structure, the rotation axis of the central body ($\mathbf{\vec{vers}_{cp}}$) and both links of leg 3 ($\mathbf{\vec{vers}_{a3}}, \mathbf{\vec{vers}_{c3}}$) have the same direction and orientation of the Z axis in the inertial frame. For leg 4 ($\mathbf{\vec{vers}_{a4}}, \mathbf{\vec{vers}_{c4}}$) the direction is the same of Z, but the orientation is symmetric.

5.3 The system of dynamic equations

From the FBD diagram (Fig.4) the dynamic equations of each element of 2D structure can be derived. For the central body (C):

$$\begin{cases} m_{cp}\frac{d^2R_x}{dt^2} = T_{x3} + T_{x4} \\ m_{cp}\frac{d^2R_y}{dt^2} = T_{y3} + T_{y4} - m_{cp}g \\ \vec{M}_{cp} = \sum_{i=3}^{4}[(\vec{R}_{bi} - \vec{R})\times\vec{T}_{yi}] \\ I_{cp}\frac{d^2\phi_z}{dt^2} = \vec{vers}_{cp}.\vec{M}_{cp} \end{cases} \tag{11}$$

For the upper link of a generic leg_i (B) (Fig.4):

$$\begin{cases} m_a\frac{d^2R_{axi}}{dt^2} = T_{jxi} - T_{xi} \\ m_a\frac{d^2R_{ayi}}{dt^2} = T_{jyi} - T_{yi} - m_a g \\ \vec{M}_{ai} = [(\vec{R}_{bi} - \vec{R}_{ai})\times(-\vec{T}_{yi})]+ \\ +[(\vec{R}_{ji} - \vec{R}_{ai})\times\vec{T}_{ji}] \\ \frac{d^2\theta_{2i}}{dt^2} = \frac{\vec{vers}_i.\vec{M}_{ai}}{I_a} - \frac{\vec{vers}_{ai}.\vec{M}_{cp}}{I_{cp}} \end{cases} \tag{12}$$

For the lower link of a generic leg_i (A) (Fig.4):

$$\begin{cases} m_c\frac{d^2R_{cxi}}{dt^2} = F_{ai} - T_{jxi} \\ m_c\frac{d^2R_{cyi}}{dt^2} = N_i - T_{jyi} - m_c g \\ \vec{M}_{ci} = [(\vec{R}_{ji} - \vec{R}_{ci})\times(-\vec{T}_{ji})]+ \\ +[(\vec{R}_{pi} - \vec{R}_{ci})\times(\vec{F}_{fi})] \\ \frac{d^2\theta_{3i}}{dt^2} = \frac{\vec{vers}_{ci}.\vec{M}_{ci}}{I_c} - \frac{\vec{vers}_{ci}.\vec{M}_{ai}}{I_a} \end{cases} \tag{13}$$

The eight last equations are generic for leg_i (i=3,4).The inertial mass of the central body, the superior link and the inferior link are respectively m_{cp}, m_a and m_c. The inertial moments I_{cp}, I_a, I_c are computed for the CM of the constituents, g is the gravitic acceleration and $\vec{\mathbf{F}}_{\mathbf{fi}}(\mathbf{F_{ai}}, \mathbf{N_i})$ (i=3,4) refers to the friction force.

Notice that the last rotation equation of (12) and (13). Beside the use of versors explained above, differential angular acceleration between two consecutive elements are computed, instead of absolute acceleration referred to the semi-positive X axis. This is done as a way to avoid more equations and variables. The body rotation angle is the same in the kinematic and dynamic modeling.

For 2D structure example 20 dynamic equations are obtained. In previous sections more 25 kinematic equations were derived to compute the auxiliary points ($\vec{\mathbf{R}}_{\mathbf{ai}}, \vec{\mathbf{R}}_{\mathbf{ci}}, \vec{\mathbf{R}}_{\mathbf{bi}}, \vec{\mathbf{R}}_{\mathbf{ji}}, \vec{\mathbf{R}}_{\mathbf{pi}}$) and the rotation versors ($\vec{\mathbf{vers}}_{\mathbf{cp}}$, $\vec{\mathbf{vers}}_{\mathbf{ai}}$, $\vec{\mathbf{vers}}_{\mathbf{ci}}$) in function of the seven independent kinematic variables. If applied external forces are known the dynamic description of the system is concluded.

5.4 Foot-soil interaction

There are a wide variety of foot soil interaction modeling. The model that is used considers a rigid surface whose shape is described in the inertial frame by $y = \varrho(x)$. The friction forces $\vec{\mathbf{F}}_{\mathbf{f}}(\mathbf{F_a}, \mathbf{N})$ are evaluated with the help of two coefficients, μ_S and μ_D, dependent of the soil features. If the tip doesn't slide along the surface than $F_a \leq \mu_S \times N$, else $F_a = \mu_D \times N$. Usually $\mu_S \geq \mu_D$. In each instant of time the leg must be in one of three states: non contact(ST3), contact without sliding(ST2), and contact with sliding(ST3).

ST1	**ST2**	**ST3**
$N_i = 0$	$R_{pxi} = a$	$F_{ai} = \mu_D.N_i$
$F_{ai} = 0$	$R_{pyi} = b$	$R_{pyi} = \varrho(R_{pxi})$

If the leg is in state ST1 then $R_{pyi} > \varrho(R_{pxi})$ and (R_{pxi}, R_{pyi}) are the tip coordinates. The leg is raised and no forces are exerted on the tip. The first column equations are added to the global system of equations. When the leg is in contact with the soil at point (a,b) two cases can be considered. If it doesn't slip along the surface (state ST2) than friction force can't be directly computed. However tip remains in the same position whose coordinates are known. Second column equations are added to the global system and determination of (N_i, F_{ai}) becomes possible. If the computed value of F_{ai} is higher than $\mu_S.N_i$ it means that the static friction force is not enough to keep the tip in (a,b). In this case the leg is in state ST3. Third column equations, instead of second column, are added to the global system whose solutions are recalculated.

5.5 Mathematical resolution

Derived a system with the same number of variables and independent equations (non singular), solutions must be determined. Consider a period of time Δt and make the discrete sampling of the variables. A system of non-linear discrete equations is

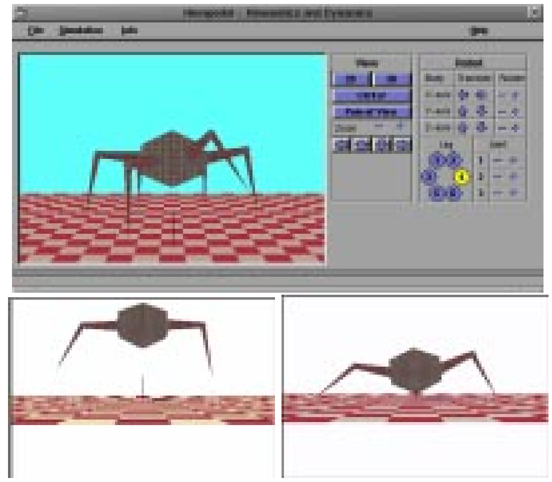


Figure 6: The hexapodal simulator. Positions of the structure during simulation

obtained by replacing the second order derivatives by a discrete approximation. In each moment $t = k.\Delta t$ system solution gives the applied forces ($\vec{\mathbf{T}}_{\mathbf{i}}, \vec{\mathbf{T}}_{\mathbf{ji}}, \mathbf{etc}$) and predicts the position of the structure in the next moment($R_x[k+1], R_y[k+1], \phi_z[k+1], etc$). Newton method is used to solve the dynamic system of equations for each instant of time.

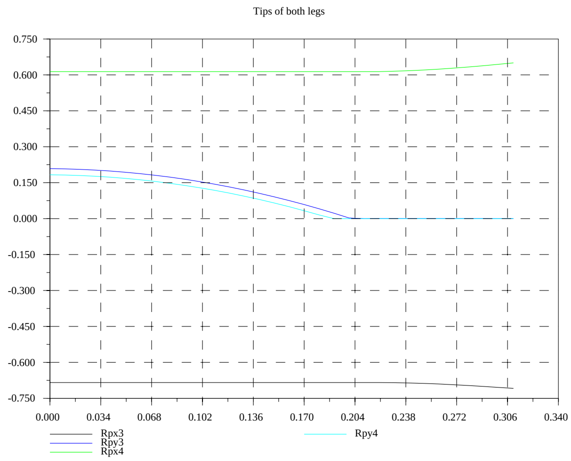
6 Simulation and results

A kinematic and dynamic simulator was programmed using the derived models (Fig.6). In this section the results of one of the many realized experiments is presented.

R_x	R_y	ϕ_z	θ_{23}	θ_{33}	θ_{24}	θ_{34}
0	0.6	0	0	245	0	255

Consider that structure falls from a starting position (Fig 6). The initial values of independent kinematic variables are in the table. The considered period of time is 0.01s and the friction coefficients are $\mu_S = 0.4$ and $\mu_D = 0.3$. Note that for t=0 the structure is not in contact with the ground.

To illustrate the studies that can be realized using the simulator, Fig.7 depicts the evolution of tips position(XY) and applied friction forces. Note that leg 4 touches the ground 0.02s before leg 3. The first one reaches the soil 0.20s after the start of motion. Around moment 0.24s both legs start slipping.



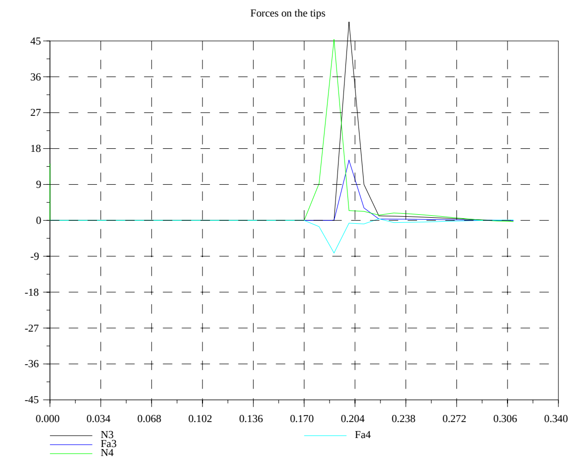


Figure 7: Graph1: Evolution of tips position of both legs in inertial frame (Position(m) versus Time(s)). Graph2: Applied forces on the tips of both legs (Force(N) versus Time(s)).

7 Discussion and conclusions

Dynamic modeling applying Lagrange Formalism is useful when a state space model is intended. However, for the example of the 2D structure, due to the complexity of the derived expressions, the symbolic inversion of matrix **D** ([1][2]), when possible, needs a huge amount of computation time. This computation is unpracticable. The option is to make the discrete sampling of the variables, as a way to obtain a set of non-linear discrete equations that can be solved with Newton method (as done in FBD). Thus, neither using FBD nor Lagrange Formalism, the state space equations are obtained. The mathematical resolution of Lagrange equations is similar to the FBD method. The Lagrange dynamic description is more condensed than FBD description. On the other hand the FBD equations are intuitive, easy to derive and allow the computation of internal forces and moments.

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