Visual Inspection and 3D Reconstruction Based on Image-to-Mirror Planes

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Abstract— **This article addresses the problem of recovering a tridimensional scene from only one uncalibrated static camera and images from planar mirrors. We present a linear solution for 3D Euclidean reconstruction based on multiocular geometry, epipolar geometry and projective geometric properties. From the recovered points of the scene we can obtain information related with distances, areas, curvatures, dimensions and even volumes of its objects. A system based on this technique could be applied on the industry, in tasks of great importance, such as the quality tests and metrology inspection. The system is simple, low cost and could achieve good performance.**

I. Introduction

It is clear that in the absence of more information, one point in the image could correspond to an infinity of points in the tridimensional scene. So it is necessary more than one image, taken from different viewpoints, to recover the 3D structure of the scene. The most important issue in reconstruction tasks, is the geometric related information between images. In this context, we could classify into three different groups of reconstruction: the projective, the affine and the Euclidean.

In the first group, the reconstruction is defined up to a projective transformation. The obtained information has no knowledge of distances, angles or parallelism. Mohr et al [6] was one of the first to use the projective geometry properties to avoid the cameras calibration. Their approach is based on the re-projection of the image points in two planes of the scene, using the invariant property of the projective co-ordinates. Six known points of the scene are the reference points, and all the recovered points are referred to this set. Faugeras [7] developed a linear method for projective reconst ruction based only in correspondent points of the two images. He chooses five arbitrary points in the scene as points of a projective base. With this choice only two parameters of the 22 that composed the two projection matrices are need to be determinate (11 parameters for each projection matrix with at least one scale factor, 2 cameras). The two unknown variables are obtained if the epipoles are known, and thus the reconstruction of all points is made. This reconstruction has no metric or affine information. At the same time, Hartley [8] obtain a similar result.

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In the affine reconstruction, defined up to an affine transformation, the obtained information has knowledge of distances and parallelism. One of the marks in the 3D affine reconstruction, done by Koenderink and Doorn [1], shows that under the parallel projection (affine transformation) the 3D structure of the scene could be recovered based only in two images. Under the orthographic projection, Tomasi and Kanade [2], obtain a similar result a nd also the rotation between the cameras. This method needs the knowledge of the intrinsic parameters of the cameras. Poelman and Kanade [3] proposed a variation of this method, in which they use the paraperspective projection to obtain a better reconstruction. In spite of its simplicity, this reconstruction is not the best model of a real camera. Thus, to treat in a better way the reconstruction problem, a lot of researchers started to use the projective geometry. Sparr [5], that developed a descriptor to the 3D affine structure of a set of points. This descriptor is independent of the affine base and its intrinsic properties, and it is link to the perspective projection of the points.

Finally, the Euclidean reconstruction, the richest one and the same as the one the humans manipulate. One method, studied by D. Weinshall [4], use in the first phase the Koenderink and Doo rn process and, from there, recover the 3D Euclidean structure using four no coplanar points. This process has the advantage of none of the knowledge of the intrinsic parameters of the cameras be necessary.

In this paper we present a 3D-reconstruction methodology, based on one uncalibrated static camera that acquires bidimensional images projected on mirrors with no related geometric information. The images from the mirrors are equivalent to images captured by cameras with different viewpoints. Our study, situated in the Euclidean 3D reconstruction, creates a linear process based on epipolar geometry and projective geometric properties. In our process we do not need the intrinsic camera parameters. All the mentioned studies, including ours, have a common point, the utilisation of a relative reference in the scene.

II. The image acquisition

To achieve results of a 3D reconstruction it is known that it is necessary at least two images. Having in mind the problem of the cost of a system that uses several cameras, we exploit the potential of mirrors to reduce its number (a mirrored image of a scene is equivalent to a virtual camera behind it). As it can be seen in the figure 1(a), we used two

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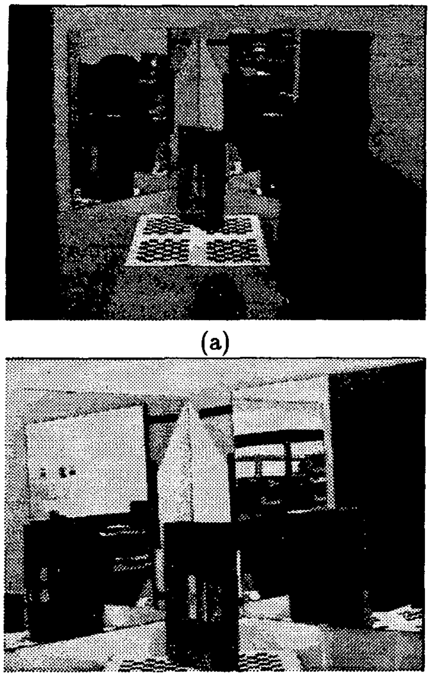


(a)

(b)

Fig. 1. *(a) A perspective of the system. (b) The image obtained with o nly one shot.*

mirrors behind the scene, that we want to recover, and one camera. We acquire, in only one shot, three parts concerning to the projection of the 3D scene and the projection of the reflection of the previous in the left and right mirror, as it can be seen in figure 1(b). Each part could be consider a different image, simulating, thus, a trinocular geometry of three cameras (two virtual and one real) observing the same scene from different viewpoints.

III. Matching

Since the reconstruction of 3D points is similar to a triangulation method, we need to have, at least, two points in the image, equivalents to the projection of the same 3D point. The process of point matching is known as correspondence and to obtain a fast solution for the correspondence problem, we compute the image geometry using two phases.

The first one is the computation of the epipoles by "construction". Having two camer as looking to the same scene, the epipole of one image is the projection of the optical centre of the other image in the previous one. The epipole is a unique point of the image that belongs to all epipolar lines. Those lines are the projections, onto images, of the epipolar planes, formed by, at least, two optical centres of both cameras and the 3D point of the scene. Every 3D point on the epipolar plane has the same epipolar line on the image. In this phase we use two correspondent points of each image.

In the second phase, is the goal of the reconstruction process that established the type of the correspondence. If the goal is to recover simple planes in the scene, the co rrespondence is established based on estimated collineations between three parts of the uncalibrated image. To reconstruct a general scene, we determine the epipolar geometry. The epipolar geometry captures all the 3D information available from the scene in one matrix, which is the fundamental matrix [7].

For the estimation of the fundamental matrices, instead of computing the 3×3 matrices by a numerical process, we use the epipole positions, obtained in the first phase. Geometric construction, and, at least, a set of two matched points in each part obtain those epipoles. The simple equation which allows to estimate those matrices, deduced in sub-section III-A, uses the geometric relation between the homography and the fundamental matrix [9]. This way, an accurate fundamental matrix, of rank 2, with a stable linear computation, is obtained with only two matched points in the image.

A. Background

Along to this explanation we will consider three generic correspondent homogeneous points,

$$\mathbf{p}_1 = \begin{bmatrix} sx_1 & sy_1 & s \end{bmatrix}^T \quad \mathbf{p}_2 = \begin{bmatrix} kx_2 & ky_2 & k \end{bmatrix}^T$$

$$\mathbf{p}_3 = \begin{bmatrix} rx_3 & ry_3 & r \end{bmatrix}^T$$

The points $\mathbf{p}_1$ and $\mathbf{p}_2$ are the reflections of a points $\mathbf{P}$ in left and right mirror and $\mathbf{p}_3$ its projection in the real image , with s, k and r scale factors.

A transformation between two image planes is a collineation called homography, represented by a 3×3 matrix. To estimate this matrix is needed at least 4 correspondent points (since the matrix depends from a scale factor, it has only 8 unknowns). In our case, since we need to relate the three 2D image parts, we must have a set of four correspondent points in each part. For each pair of points we establish the following equation system

$$\alpha \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_i = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_j \qquad (1)$$

with α an arbitrary factor, and i and j the index of the images with $i \neq j$. To obtain all the homographies between images (real and reflected), we only need to estimate two, of the three homographies. Since all homographies are a 3×3 invertible matrices, from equation (1) we have

$$\mathbf{p}_1 = \mathbf{H}_{13}\mathbf{p}_3 \Leftrightarrow \mathbf{p}_3 = \mathbf{H}_{13}^{-1}\mathbf{p}_1 \quad \textit{and} \quad \mathbf{p}_2 = \mathbf{H}_{23}\mathbf{p}_3 \qquad (2)$$

The fundamental matrix, $\mathbf{F}$, could be decomposed by

$$\mathbf{F} = [\mathbf{e}_2]_\wedge^T \mathbf{H} = -[\mathbf{e}_2]_\wedge \mathbf{H}$$

The same analysis could be done for the reflected left image, from where we obtain

$$\mathbf{F} = -\mathbf{H}^{-T}[\mathbf{e}_1]_\wedge^T = \mathbf{H}^{-T}[\mathbf{e}_1]_\wedge$$

with $\mathbf{e}_1$ the epipole of the reflected left image.

It is possible, creating a projective base in each image with a set of correspondent points, reduce the number of the parameters that compose the homography [9]. Thus, supposing, for each image, four points such

$$\mathbf{p}_a = [\ 0 \quad 0 \quad \gamma\ ]^T \quad \mathbf{p}_b = [\ \alpha \quad 0 \quad 0\ ]^T$$

$$\mathbf{p}_c = [\ 0 \quad \beta \quad 0\ ]^T \quad \mathbf{p}_d = [\ \tau \quad \tau \quad \tau\ ]^T$$

If the points are images from points in a plane, those images are related by the collineation

$$\mathbf{p}_{i(1stimage)} = \lambda_i \mathbf{H} \mathbf{p}_{i(2ndimage)}$$

with $i = a, b, c, d$, $\mathbf{H}$ the homography and λ_i arbitrary factors. Each set in the image form a projective basis, i.e. no three of them are collinear. Using the previous equation with the first three matched pairs, we obtain the general equation for $\mathbf{H}$, given by

$$\mathbf{H} = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{bmatrix} \tag{3}$$

Since the collineation is defined up to a scale factor, the above matrix has only two independent parameters. Furthermore, this three parameters are non zero for a non-singular homography. Since our homography maps a plane onto a plane, $\mathbf{H}$ is not singular and we can fix $\gamma = 1$. From this result, if we know the epipoles, we could estimate the fundamental matrix with only two points.

B. First phase - Epipoles by construction

To obtain the epipole we could use the intersection of two epipolar lines or compute it by the null space of the fundamental matrix. Since we are using mirrors, the epipoles can be obtained directly from the intersection of epipolar lines (without computing the fundamental matrix). Thus, a generic 3D point and its mirror reflection belong to the same projection plane.

Observing our uncalibrated images, we realise that we have projections of 3D points and its reflections, from where we could form two epipolar lines. The epipoles are obtained by the intersection of the epipolar lines (at least two), geometrically constructed, from the refereed matched points.

Analytically, let be $\mathbf{p}_3$ the real projection, in the image, of the 3D point $\mathbf{P}$ and $\mathbf{p}_1$ the reflection of projected point of $\mathbf{P}$ in the same image. The line formed by those points is given by

$$(\mathbf{p}_1 \wedge \mathbf{p}_3)^T \mathbf{q} = 0$$

with $\mathbf{q}$ a generic point of the line and s, w and r scale factors. Having a set of n correspondent points ($n \geq 2$), we form a linear system $\mathbf{Ax} = \mathbf{b}$, with n equations and *two variables. Comparing the equations we easily conclude* that the only solution for the created system must be the epipole. This calculus has to be done twice to obtain the two epipoles on each reflected image part. This linear system is solved by the least square method using singular value decomposition [10].

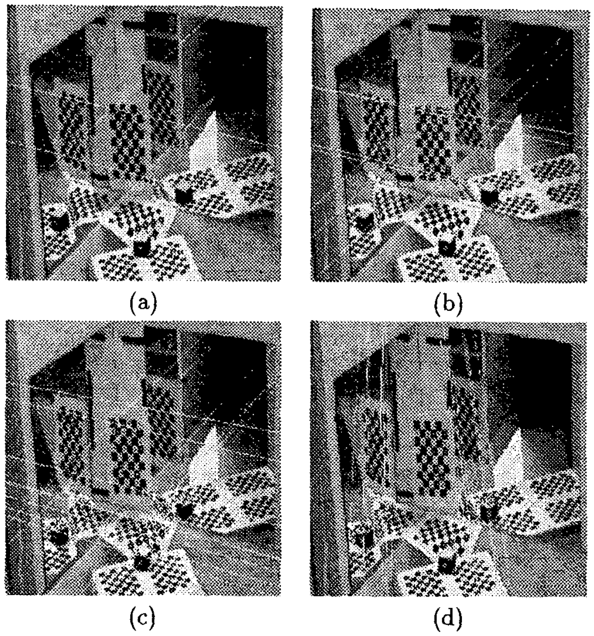


Fig. 2. *Lines formed to obtain the epipoles by its intersection: (a) Two points case; (b) Four points case; (c) Eight points case; (d) Epipolar lines obtain by the fundamental matrix computed with an eight points algorithm[12].*

The figure 2 shows graphicaly the lines which were intersect to obtain the results presented in table 1. In table 1, we could see the epipoles, of the reflected parts of the uncalibrated image, of four different situations. The first three cases of that table use our graphical method, while the last one, estimate the fundamental matrix and then calculate the null space, which give the epipoles co-ordinates. In a comparison of both methods, we high light the fact that our method could obtain the epipoles with only a pair of correspondent points in each image part (first case) while the other needs more points to compute the fundamental matrix and achieved the same result (last case). The number of points in this calculus is very important because affects noise sensitivity. As our method is a pure graphic calculation, the sensitive to the noise became softer, but does not disappear. Analysing the three first cases of table I, we notice that with the increment of the points used the noise it is reduced, became the epipoles values more stable. Comparing the two last cases, it could be seen the great difference between the epipoles co-ordinates. This happen because the fundamental matrix was estimated with the minimum number of the points, having a high level of sensibility to noise. It is important to focus that the signal of the epipoles co-ordinates is the same for all cases.

Number of used points	*Left epipole*	*Right epipole*
2	(-6974.3,-871.3)	(1910.2,-801.0)
4	(-2225.0,-098.0)	(1801.6,-707.3)
8	(-2959.7,-224.6)	(1629.0,-616.2)
8	(-0878.6,-591.2)	(0186.2,-525.3)

TABLE I

Results of the epipoles calculation. The first three cases use our graphical method. The fourth case uses the eight-point algorithm for the fundamental matrix estimation, followed by the null space calculation[12]. It is consider the top-left point of the image as origin.

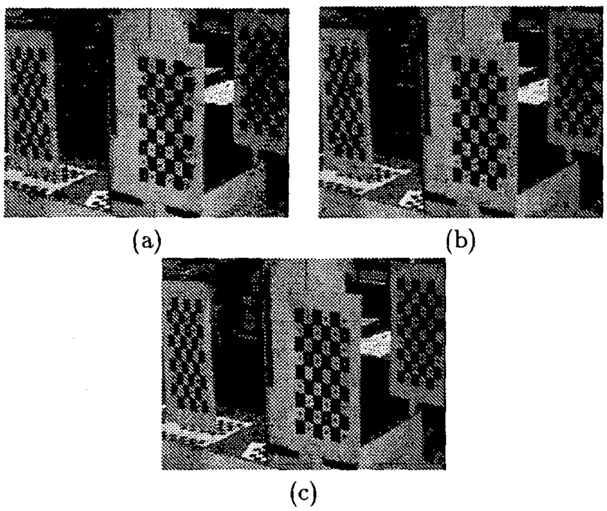


Fig. 3. *Representation of the points obtained by the use of the estimated homographies: (a) Four points case; (b) Eight no normalised points case; (c) Eight normalised points case.*

C. Second phase - Homographies and Fundamental matrices

To achieve the final goal of the reconstruction process, the correspondence between points must be established. To recover simple planes in the scene, we estimate collineations between the three parts of the uncalibrated image. To reconstruct some complex structures, we determine the fundamental matrix.

The computation of homographies between planes, for a planar reconstruction, begins by choosing a set of four points in two different parts of the image. Then the entire set is submitted to a normalisation process to have better balanced data and increase the estimation performance [12]. The two linear systems, each one based on one equation of (2), are solved by the least square method using the eigenvalues decomposition [10].

Table 2 presents the results of three cases, of all involved homographies estimation for the image. In figure 3 are shown, graphically, all used points in the estimations (marked by a cross) and all points resulting by the homographies (marked by a circle and a plus). Analysing the results of the table 2, we could notice that with the entry data normalisation the results became better (the second case of table 2 has worst median and standard deviation of the sum of distances between all points and its projections than the third). Another fact, which return by comparing the first and last situation of table 2, is that the use of more exact matched points increase the performance of this correspondence (the standard deviation of the last case is the better). As explained above, in those estimations, the increment of exact correspondent points decrease the noise sensibility.

The estimation of the fundamental matrix, $\mathbf{F}$, must satisfy the epipolar and rank 2 constraint. This estimation is based on the estimation of the epipoles by "construction" from the first phase. Using epipole co-ordinates and equation (3) (with $\gamma = 1$) we obtain

$$\mathbf{p}_2^T \begin{bmatrix} 0 & 1 & -e_{y2} \\ -1 & 0 & e_{x2} \\ e_{y2} & -e_{x2} & 0 \end{bmatrix} \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{p}_1 = 0$$

with two variables, α and β. Having n correspondent points ($n \geq 2$), we create an equation system formed by n linear equations. The solution of this system give us the values α and β, that defines $\mathbf{F}$. The system was solved by the least square method using the eigenvalues decomposition [10].

The table 3 presents some results of the estimation of all involved fundamental matrices for the uncalibrated image and normalisation. From the mean distance it could be seen that with the increment of the number of matched points, we achieved better results. Comparing table 1 with table 3, it could be noted that the epipoles co-ordinates are identic but not exact. This happen because our estimation process, in spite of more robust than the usual, still sensitive with noise. The solution to reduce this problem pass by the noise filtering of the entry se ts with the RANSAC technique in the estimation of α and β [11].

Figure 4 presents the application of the estimated fundamental matrices, which are the epipolar lines. Comparing this figure with figure 2 it is high light the fact that it is evident that epipolar geometry was estimated based on the epipoles.

IV. 3D Reconstruction

In this step, we recover the 3D information of a scene point using geometric intersection of a pair of rays, both passing by the 3D point and by the respective camera's optical centres. The intersection of those rays is the solution $\mathbf{P} = (X, Y, Z)$. The equation for these rays is

$$\mathbf{P} = \mathbf{C}_i + \lambda \frac{(\mathbf{P}_i - \mathbf{C}_i)}{||\mathbf{P}_i - \mathbf{C}_i||} \qquad i \in \{1,2,3\} \ \textit{and} \ \lambda \neq 0 \quad (4)$$

Number of points	Points normalised	H_{13}	H_{23}	H_{12}	H_{12a}
		Mean distance between all points and its reprojections			
4	Yes	1.9302 pixels	1.2765 pixels	1.2963 pixels	1.2963 pixels
8	No	4.8961 pixels	1.8563 pixels	3.3249 pixels	1.3504 pixels
8	Yes	2.0119 pixels	1.4386 pixels	1.1932 pixels	1.2385 pixels
		Standard deviation of the distance between all points and its reprojections			
4	Yes	2.4350 pixels	1.6361 pixels	1.4294 pixels	1.4294 pixels
8	No	2.6860 pixels	0.7665 pixels	1.7170 pixels	0.7532 pixels
8	Yes	0.6505 pixels	0.6907 pixels	0.4949 pixels	0.5728 pixels

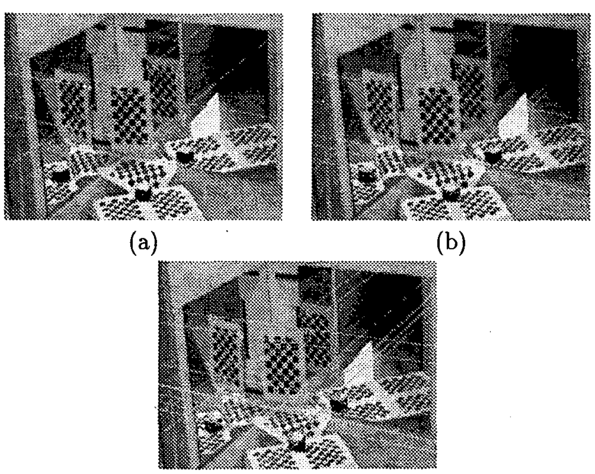
TABLE II

Results of homographies estimation. The H_{12a} matrix was estimated with the correspondent points, while H_{12}.

No. points	Fundamental matrices	Mean distance	α and β	Epipoles (from null space of F)
2	$\mathbf{F}_{13} = \begin{bmatrix} 0 & -1.0 & -871.3 \\ 1.0 & 0 & 6974.3 \\ 871.3 & -6974.3 & 0 \end{bmatrix}$	$Q_{f13} = 2.0985$	$\alpha_{13} = 1.0$, $\beta_{13} = 1.0$	$\mathbf{e}_1 = (-6974.3, -871.3)$
	$\mathbf{F}_{23} = \begin{bmatrix} 0 & -1.0 & -942.3 \\ 1.0 & 0 & -2107.2 \\ 942.3 & 2107.2 & 0 \end{bmatrix}$	$Q_{f23} = 1.3938$	$\alpha_{23} = 1.0$, $\beta_{23} = 1.0$	$\mathbf{e}_2 = (2107.2, -942.3)$
4	$\mathbf{F}_{13} = \begin{bmatrix} 0 & -1.1 & -486.5 \\ 1.0 & 0 & 4273.0 \\ 459.7 & -4228.4 & 0 \end{bmatrix}$	$Q_{f13} = 1.4482$	$\alpha_{13} = 1.0584$, $\beta_{13} = 1.0106$	$\mathbf{e}_1 = (-4228.4, -459.7)$
	$\mathbf{F}_{23} = \begin{bmatrix} 0 & -1.0 & -708.8 \\ 1.0 & 0 & -1760.1 \\ 721.0 & 1723.7 & 0 \end{bmatrix}$	$Q_{f23} = 1.1366$	$\alpha_{23} = 1.0172$, $\beta_{23} = 0.9793$	$\mathbf{e}_2 = (1730.3, -723.7)$
8	$\mathbf{F}_{13} = \begin{bmatrix} 0 & -1.0 & -231.0 \\ 1.0 & 0 & 2974.6 \\ 224.6 & -2959.7 & 0 \end{bmatrix}$	$Q_{f13} = 0.7201$	$\alpha_{13} = 1.0281$, $\beta_{13} = 1.005$	$\mathbf{e}_1 = (-2959.7, -224.6)$
	$\mathbf{F}_{23} = \begin{bmatrix} 0 & -1.0 & -616.2 \\ 1.0 & 0 & -1629.7 \\ 620.6 & 1614.2 & 0 \end{bmatrix}$	$Q_{f23} = 1.1567$	$\alpha_{23} = 1.0072$, $\beta_{23} = 0.9909$	$\mathbf{e}_2 = (1617.3, -621.8)$

TABLE III

Results of the estimation of the fundamental matrices. The mean distances and epipole co-ordinates are in pixels. The fifth column is achieved after the estimation of the matrices, by calculate the null space of those matrices.



(a) (b) (c)

Fig. 4. *Epipolar lines of the estimated fundamental matrices: (a) Two points case; (b) Four points case; (c) Eight points case.*

Has we could see from equation (4), we must have four 3D points to accomplish the reconstruction. Two of them are the optical centres of camera i, $\mathbf{C}_i$, while each point $\mathbf{P}_i$ belongs to a ray i. To obtain the points $\mathbf{P}_i$ and $\mathbf{C}_i$, we will use the matrix that relates projected points into Euclidean points in a plane (the planar mirrors) on world co-ordinate system.

Since we are using planar mirrors, we assume without losing generality, that one of the mirrors represents the XZ plane and the other the YZ plane. This fact brings to us an easy way to compute the mentioned transformation matrix, $\mathbf{T}$, for that specific mirror and its related image part. The 3D mirror point $\mathbf{P}_i$, corresponding to the intersection of the projective ray with planar mirror, is related with its image by

$$\mathbf{P}_i = \mathbf{T}^{-1}\mathbf{p}_i \qquad i \in \{1, 2\} \tag{5}$$

in homogeneous co-ordinates. This ray also passes through the camera's optical centre, $\mathbf{C}_i$.

Obtained $\mathbf{C}_1$ and $\mathbf{C}_2$, with the intersection of the lines that pass by the optical centre and the 3D points in each

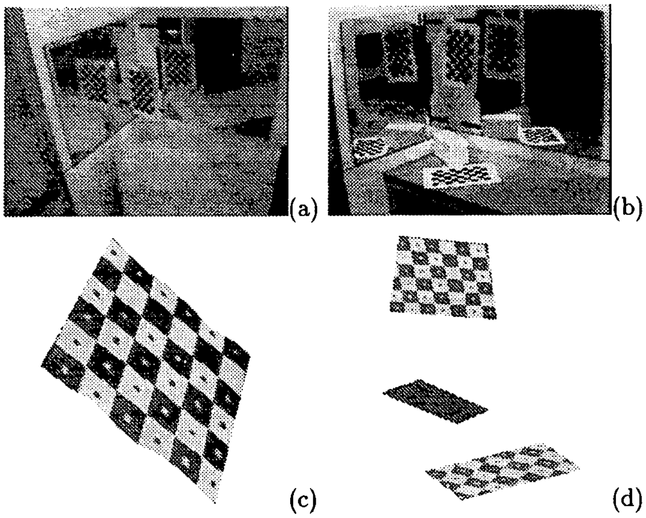


Fig. 5. *(a) First 2D uncalibrated image from where we recover; (b) Second 2D uncalibrated image with the points that w e recover; (c) The results of the 3D planar reconstruction based on image (a); (d) The results of the 3D planar reconstruction based on image (b).*

mirror plane and equivalent to the intersection of image rays with the planar mirrors, we could recover the 3D point. The reconstructed point $\mathbf{P} = (X, Y, Z)$, using the camera's optical centres $\mathbf{C}_i = (C_{Xi}, C_{Yi}, C_{Zi})$ and the mirror's intersection points $\mathbf{P}_i = (X_i, Y_i, Z_i)$, is the solution of the equation

$$\mathbf{P} = \mathbf{C}_i + \gamma \frac{(\mathbf{P}_i - \mathbf{C}_i)}{||\mathbf{P}_i - \mathbf{C}_i||} \Leftrightarrow A \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = b$$

with $i \in \{1, 2\}$. Experiments with real images validate our simple reconstruction method, from where we present some results.

Each figure 5(a) and figure 5(b), represent uncalibrated images of 3D planes that we want to recover by the method reported before. The obtained points from the image are the points that we recover to interpolate the surfaces. The results of that interpolation are shown in figure 5(c) and figure 5(d). To give a better view of the accuracy of the reconstruction, we present some numerical results of the case represented by figure 5(a) and figure 5(c), like the distances between the real and recovered point and the distances between the neighbourhood reconstructed points.

For this particular example, the mean errors between the 3D real and recovered co-ordinates are $\Delta X = 12.31006667$, $\Delta Y = 4.0906875$ and $\Delta Z = 3.9457625$ millimetres. With $(192, 103, 353)$ and $(207, 228, 152)$, two known 3D points of the scene, the estimation of the 3D centres in millimetres, are

$$\mathbf{C}_1(Left\ Virtual\ Centre) = (1210.6, -739.0, 718.7)$$
$$\mathbf{C}_2(Right\ Virtual\ Centre) = (-1490.1, 1094.9, 857.8)$$
$$\mathbf{C}_3(Real\ Centre) = (1082.4, 852.4, 831.4)$$

V. Conclusion

Our method treats the planar reconstruction with excelent results. We are now dealing with dense reconstruction of the scene, where we are developing a technique to automatically increase the number of correspondent points in the three images. This technique could use more mirrors in adequate positions, to maintain the goal of build a low cost and robust reconstruction system. Our effort is to develop a simple and robust automatic process that with no doubt could extract the match points on each image. With those, we can create, at a low cost, a robust system to recovery the entire 3D structure of the scene where are including the occult parts.

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