Abstract—This paper addresses the stability and tracking control problem of quadrotor unmanned flying vehicle (UAV) in the presence of uncertainty. Adaptive autonomous sliding mode tracking system is designed by combining robust and adaptive control theory. Lyapunov analysis shows that the proposed algorithms can guarantee asymptotic convergence of the tracking error of the linear and angular motion dynamics of the vehicle. Compared with other existing adaptive backstepping design, the proposed method is very simple and easy to implement on an actual system as it does not require multiple design steps without augmented signals and a priori known bound of the uncertainty. To illustrate the theoretical evaluation, simulation results on custom made quadrotor UAV system are presented for real-time applications.

Index Terms—Quadrotor Aerial Vehicle; Lyapunov Method; Adaptive Control.

I. INTRODUCTION

Recently, more and more researchers and industrial companies have focused their attention on designing a new type of small scale unmanned aerial vehicles. The interest in such small scale vehicles is growing in military and civilian applications, such as surveillance, inspection, search and rescue missions in dangerous or hostile environment. The design of autonomous flight control system for small scale quadrotor UAV in the presence of uncertainty is very difficult tasks due to its inherent nonlinearity associated with the dynamical model, underactuated property and external disturbances associated with uncertain flying environment. Over the past years, various automatic flight control systems for quadrotor system have been reported in the literature [2-6, 8-20]. Among these designs, PID and LQR type classical control mechanism has been widely used for commercial quadrotor system [1], [4], [5], [8], [10], [11], [19]. These classical algorithms may exhibit poor hovering performance because of the modeling error uncertainty. Backstepping control technique has been proposed to address the problem associated with the modeling error dynamics of the vehicle in [12, 15, 16, 17, 18]. Later, in [13], author included integral action with the backstepping technique in order to minimize the steady state tracking error. Most quadrotor unmanned flying robots are usually very small and lightweight, making the system sensitive to the variation in payload and uncertainty. This means that additional payload mass, moment of inertia, aerodynamic and gyroscopic force may change vehicle dynamics, affecting the stability and tracking performance significantly. Furthermore, unpredictable changes in flying environment may increase the modeling error and uncertainty, making the flight control system design even more complicated. As a result, available classical flight control system may not be able to deal with the change in flight dynamics for different flight missions. Therefore, the problem associated with uncertainty remains a challenging task demanding advanced autonomous flight control design for quadrotor UAV system. To deal with above mentioned problem, nonlinear control technique has been employed for designing autonomous quadrotor UAV system in [2], [3], [6], [9], [20]. In [2], authors have proposed so called dynamic inversion mechanism for hovering flight control system design by using well-known feedback linearization technique. $H_\infty$ control technique combined with backstepping control mechanism in [6]. Nonlinear adaptive control algorithms using the backstepping technique proposed in [9]. Most recently, adaptive backstepping control algorithm technique used to design nonlinear control for quadrotor UAV systems [3, 21]. However, the design and implementation mechanism of existing nonlinear control algorithms are very complicated as they usually associated with augmented auxiliary signals requiring multiple design and computation steps. So, our aim in this work is to develop very simple nonlinear adaptive flight control strategy which can cope modeling error and disturbances uncertainty. In this paper, we propose adaptive sliding mode control for stability and tracking control of quadrotor vehicle in the presence of uncertainty. Virtual adaptive position control algorithm combines gravity compensation with the desired linear acceleration and proportional-derivative error like terms. Adaptive attitude controller comprises proportional-derivative error like term with the desired angular acceleration term. Adaptation laws are used in both position and attitude dynamics to learn and compensate uncertainty associated with the variation of the payload mass, inertia, aerodynamic and gyroscopic force, external disturbances and unpredictable change in outdoor flying environment. Lyapunov method is employed to develop control algorithm and to analyze the convergence property of the linear and angular state dynamics. Unlike existing methods, the design does not use augmented signals and multiple steps which makes the design very simple and easy to implement for practical applications. Moreover, the design does not require a priori known bound of the uncer-
tainty. Finally, various evaluation results on our laboratory
made miniature quadrotor aerial vehicles are presented to
demonstrate the effectiveness of the proposed method for
practical applications.

II. MODEL DYNAMICS, ALGORITHM DESIGN AND
STABILITY ANALYSIS

We first derive mathematical model of the quadrotor
flying vehicle [12, 13]. The position of the vehicle is
defined as \( P(t) = [x(t)\ y(t)\ z(t)]^T \) and its attitude represented
by three Euler angles as \( \Theta(t) = [\phi(t)\ \theta(t)\ \psi(t)]^T \).
The three translational and rotational velocities are defined as
\( V(t) = [V_1(t)\ V_2(t)\ V_3(t)]^T \) and \( \Omega(t) = [\Omega_1(t)\ \Omega_2(t)\ \Omega_3(t)]^T \), respectively. Then, the relationship
between velocities \( (\dot{P},\Theta(t)) \) and \( (V,\Omega) \) can be written for
earth fixed earth inertial reference frame and body fixed frame
attached to the vehicle as follows

\[
\dot{P} = R_t(\Theta)V,\ \Omega = B(\Theta)\dot{\Theta}
\]

where \( R_t \in \mathbb{R}^{3 \times 3} \) and \( B \in \mathbb{R}^{3 \times 3} \) are the transformation
velocity matrix and the rotation velocity matrix between fixed
inertial frame and body fixed frame as given as follows

\[
R_t = \begin{bmatrix}
C_\phi S_\psi & S_\phi S_\psi & C_\phi C_\psi - S_\phi S_\psi \\
C_\psi & -S_\psi & C_\phi \phi \\
S_\phi & S_\phi C_\psi - C_\phi S_\psi & C_\phi C_\psi + S_\phi S_\psi
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
1 & 0 & -S_\theta \\
0 & C_\phi & C_\theta S_\phi \\
0 & -S_\phi & C_\theta C_\phi
\end{bmatrix}
\]

where \( S(.) \) and \( C(.) \) denote \( \sin(.) \) and \( \cos(.) \), respectively.

We now take the derivative equation (1) to constitute the
kinematic equations for the quadrotor vehicle

\[
\dot{P} = R_t \dot{V} + R_t \dot{V} \\
\dot{\Omega} = B \dot{\Theta} + \left( \frac{\partial B}{\partial \phi} \dot{\phi} + \frac{\partial B}{\partial \theta} \dot{\theta} \right) \dot{\phi}
\]

Using \( \dot{R}_t = R_t S(\Omega) \) with the skew-symmetric matrix \( S(\Omega) \)

\[
S(\Omega) = \begin{bmatrix}
0 & -\Omega_3 & \Omega_2 \\
\Omega_3 & 0 & -\Omega_1 \\
-\Omega_2 & \Omega_1 & 0
\end{bmatrix}
\]

we can write equation (4) and (5) in the following form

\[
\dot{P} = R_t (\dot{V} + \Omega \times V) \\
\dot{\Omega} = B \dot{\Theta} + D(\Theta, \dot{\Theta})
\]

with

\[
D(\Theta, \dot{\Theta}) = \begin{bmatrix}
-C_\phi \dot{\phi} \\
-S_\phi \dot{\phi} + C_\phi C_\psi \dot{\phi} - S_\phi S_\psi \dot{\phi} \\
-C_\phi \dot{\phi} - S_\phi C_\psi \dot{\phi} - C_\phi S_\psi \dot{\phi}
\end{bmatrix}
\]

The dynamic equation of motion for the vehicle subjected to
forces \( U_f \) and moments \( U \) applied to the center of the mass
can be derived as

\[
\dot{P} = \beta U_f - C \dot{P} - \gamma \\
\dot{\Theta} = MU - \eta D(\Theta, \dot{\Theta}) - \xi \dot{\Theta} - B \dot{\Theta} \times B \dot{\Theta} \times B \dot{\Theta} - B \dot{\Theta} \times \sum_{i=1}^4 I_i \omega_i
\]

where \( U_f \) is the force generated by the propellers, \( U \) is the
total moments developed by the propellers, \( M = (IB)^{-1} \),
\( n = B^{-1} \), \( \beta = (mR_f^2)^{-1} \) with constant payload mass
\( m \), \( C = m^{-1} L \) with aerodynamic drag coefficients \( L = diag \{ L_d, L_d, L_d \} \) with \( L_d > 0 \), \( L_d > 0 \) and \( L_d > 0 \),
\( \gamma = TH \) with \( T = [0, 0, 1]^T \), \( H = [0 \ 0 \ g]^T \) and \( g = 9.81 m/s^2 \).
\( I_r \) is the inertia of the rotor blade, \( \omega_i \) are the angular rota-
tional velocities of the rotors and \( \xi = \Gamma^T M \) with symmetric
positive definite constant inertia matrix \( I = diag \{ I_x, I_y, I_z \} \) and
aerodynamic coefficients \( M = diag \{ M_1, M_2, M_3 \} \)
\( M_1 > 0 \), \( M_2 > 0 \) and \( M_3 > 0 \). Let us now introduce
adaptive flight control strategy for the quadrotor UAV system
given by equation (10) and (11). It is assumed that the
translational and rotational dynamics are affected by external
uncertainty disturbances as \( F_\psi(t) = [F_\psi(t), F_\psi(t), F_\psi(t)] \)
and \( F_\psi(t) = [F_\psi(t), F_\psi(t), F_\psi(t)]^T \) and
\( F_\psi(t) = [F_\psi(t), F_\psi(t), F_\psi(t)]^T \). We also assume that
the desired task \( x_{1d}, x_{3d} \) and their first and second derivatives are
bounded and belongs to a known compact set. Throughout
our stability analysis, the position, orientation and their first
derivatives are assumed to be available for measurement.
Since \(-\pi < \phi < \pi \), \(-\pi < \theta < \pi \) and \(-\pi < \psi < \pi \),
the matrices \( R_t \) and \( B \) are bounded as \( \| R_t \| \leq k_r \) with
\( k_r > 0 \) and \( \| B \| \leq k_b \) with \( k_b > 0 \). We now design adaptive
flight controller for the attitude, altitude and virtual position
dynamics such that \( (\dot{\phi}, \dot{\theta}, \dot{\psi}) \) and \( (x, y, z) \) converges to
the desired values of \( (\phi_d, \theta_d, \psi_d) \) and \( (x_d, y_d, z_d) \). To do that,
let us define the following state variables for the position
and attitude dynamics as \( x = P \) and \( x_3 = \Theta \). Then, the
error model can be presented by the following state space
equation

\[
\dot{e}_1 = e_2, \dot{e}_2 = -\beta U_f + \gamma + C x_2 - F_a + x_{1d} \\
\dot{e}_3 = e_4, \dot{e}_4 = -MU - \zeta + e_{2d}
\]

where \( U_f = U_t, \zeta = f(x, x_d) + F_p, f(x, x_d) = -\eta D(x, x_d) - \xi x_4 - B x_4 \times B x_4 - B x_4 \sum_{i=1}^4 I_i \omega_i, e_1 = (x_{1d} - x_1), e_2 = (x_{2d} - x_2), e_3 = (x_{2d} - x_3) \) and \( e_4 = (x_{3d} - x_3) \). The, we define sliding surface for linear and an-
terior dynamics as \( S_L = (e_2 + \lambda L e_1) \) and \( S_A = (e_4 + \lambda_A e_3) \) with positive definite constant diagonal matrices \( \lambda_L \in \mathbb{R}^{1 \times 3} \) and \( \lambda_A \in \mathbb{R}^{1 \times 3} \). Then, we introduce the following adaptive
control algorithm for \( U_t \)
To validate the control algorithm developed in previous section, various evaluations have been performed on

\[ \mathcal{U}_t = \beta^{-1}(\ddot{x}_d + K_c x + \lambda_L e_2 + \gamma + k_L S_L - u_t) \]

\[ u_l = \hat{\theta}_1 \text{sign}(S_L), \hat{\theta}_1 = \Gamma_1 S_L^T \text{sign}(S_L) \]  

where \( \hat{\theta}_1 = (\theta_1 - \hat{\theta}_1), \hat{\theta}_1 \) is the estimate of \( \|F_u\| \leq \theta_1, \)

\[ K_c = \text{diag}[K_{c1}, K_{c2}, K_{c3}], \lambda_L = \text{diag}[\lambda_{L1}, \lambda_{L2}, \lambda_{L3}], \]

\[ k_L = [k_{L1}, k_{L2}, k_{L3}], \Gamma_1 > 0 \text{ and } \mathcal{U}_l = [\mathcal{U}_x, \mathcal{U}_y, \mathcal{U}_t]. \]  

We now introduce the following adaptive control algorithm for \( \mathcal{U} \)

\[ \mathcal{U} = \mathcal{M}^{-1}(\ddot{x}_d + \lambda_A e_4 + k_A S_A - u_A) \]

\[ u_A = \hat{\theta}_2 \text{sign}(S_A), \hat{\theta}_2 = \Gamma_2 S_A^T \text{sign}(S_A) \]  

where \( \hat{\theta}_2 = (\theta_2 - \hat{\theta}_2), \hat{\theta}_2 \) is the estimate of \( \|\gamma_d\| \leq \theta_2, \)

\[ \lambda_A = \text{diag}[\lambda_{A1}, \lambda_{A2}, \lambda_{A3}], k_A = [k_{A1}, k_{A2}, k_{A3}], \Gamma_2 > 0 \text{ and } \mathcal{U} = [\mathcal{U}_x, \mathcal{U}_y, \mathcal{U}_t]. \]  

To avoid the discontinuity of the learning estimate, one needs to employ projection based learning estimate [22] as designed

\[ \hat{\theta}_1 = \text{Proj}(\theta_1, \Gamma_1 S_L^T \text{sign}(S_L)) \]  

\[ \hat{\theta}_2 = \text{Proj}(\theta_2, \Gamma_2 S_A^T \text{sign}(S_A)) \]  

The desired roll, \( \phi_d \), and pitch angle, \( \theta_d \), in virtual adaptive position control can be calculated from the relationship \( \phi_d = \arcsin(\mathcal{U}_x \sin(\psi_d) - \mathcal{U}_y \cos(\psi_d)) \) and \( \theta_d = \arcsin(\mathcal{U}_x \cos(\psi_d) + \mathcal{U}_y \sin(\psi_d)) \) with the given desired trajectory for yaw \( \psi_d \). For algorithm design, stability and tracking convergence analysis, we consider the following composite Lyapunov functional

\[ V = \frac{1}{2} S_L^T S_L + \hat{\theta}_1^T \Gamma_1^{-1} \hat{\theta}_1^T + \frac{1}{2} S_A^T S_A + \hat{\theta}_2^T \Gamma_2^{-1} \hat{\theta}_2^T \]  

We take the derivative (17) along the trajectories of the closed system formulated by using equation (12), (13), (14), (15) and (16). Then, \( \dot{V} \) can be simplified as

\[ \dot{V} \leq -\lambda_{\min}(k_L) \|S_L\|^2 - \lambda_{\min}(k_A) \|S_A\|^2 < 0 \]  

with \( V \in L_2 \) and \( V \in L_\infty \). Then, in view of equation (18) and Barbalat’s Lemma [14] together with projection mechanism (15) and (16), we can conclude that the error signals \( S_L \) and \( S_A \) are bounded and their bounds asymptotically converges to zero in Lyapunov sense. Since the signals \( S_L \) and \( S_A \) are bounded, then signals \( e_1, e_2, e_3 \) and \( e_4 \) are also bounded and their bounds also asymptotically converges to zero in Lyapunov sense.

III. DESIGN SYNTHESIS AND EVALUATION RESULTS

To validate the control algorithm developed in previous section, various evaluations have been performed on
Fig. 5. Time history of state dependent disturbance $d_z$ in $z$ direction in meters.

Fig. 6. External state dependent disturbance $d_{\phi}$ with $\phi$ direction in radians.

Fig. 7. Time history of $d_\theta$ in $\theta$ direction in radians.

Fig. 8. Disturbance $d_\psi$ with $\psi$ direction in radians.

Fig. 9. Desired yaw angle task $\psi_d$ (red-solid line) and actual output $\psi$ (black-dash line) in radians.

Fig. 10. Desired position tracking $x_d$ (red-solid line) and actual output $x$ (black-dash line) in meters.
Fig. 11. Desired position tracking $y_d$ (red-solid line) and actual output $y$ (black-dash line) in meters.

Fig. 12. Desired altitude tracking $z_d$ (red-solid line) and actual output $z$ (black-dash line) in meters.

Fig. 13. Desired rolling angle $\phi_d$ (red-solid line) and actual output $\phi$ (black-dash line) in radians.

Fig. 14. Desired pitch angle $\theta_d$ (red-solid line) and actual output $\theta$ (black-dash line) in radians.

our laboratory made quadrotor UAV system as shown in Fig. 1. These evaluation results are based on the dynamic model presented by equation (10) and (11). In our evaluation, the desired trajectories for $x_d$, $y_d$ and $z_d$ are chosen as $x_d(t) = \left(1 - e^{-5t^3}\right) \sin(10t)m$, $y_d(t) = \left(1 - e^{-5t^3}\right) \cos(10t)m$, $z_d(t) = \left(1 - e^{-5t^3}\right)m$. The reference attitude angles $\psi_d$ are generated by using the following transfer function $G(s) = \frac{1}{s}$. The desired roll, $\phi_d$, and pitch angle, $\theta_d$, is generated from the relationship $\phi_d = \arccos(Ux\sin(\psi_d) - Uy\cos(\psi_d))$ and $\theta_d = \arcsin\left(Ux\cos(\psi_d) + Uy\sin(\psi_d)\right)$. Implementation diagram of the proposed adaptive sliding mode control algorithm is depicted in Fig. 2. The parameters of the the vehicle are selected as payload mass $m = 5$ kg, distance from the center of the mass to the rotor axes $d = 0.2$ m, lift constant $\alpha = 3.5 \times 10^{-5} \frac{Nm}{rad^2}$, $g = 9.81 \frac{m}{s^2}$, the drag factor $\alpha_d = 0.0032 \frac{Nm}{\frac{m}{rad^2}}$, $I_x = 2 Nm\cdot rad$, $I_y = 3 Nm\cdot rad$, $L_1 = 4 N\cdot \frac{m}{rad}$, $K_{d1} = 5 N\cdot \frac{m}{rad}$, $L_{d3} = 6 N\cdot \frac{m}{rad}$, $M_1 = 3 Nm\cdot rad$, $M_2 = 5 Nm\cdot rad$, and $M_3 = 3 Nm\cdot rad$. To analyze the robustness of the proposed design, we choose the mass and inertial parameters four times larger than the actual values of the vehicle making large modeling error uncertainty. For our evaluation, the state dependant disturbances for the translational and rotational dynamics are chosen as $F_x = y\sin(pit) + z\sin(pit)$, $F_y = z\sin(pit) + x\cos(pit)$, $F_z = y\sin(pit) + x\cos(pit)$, $F_\phi = \delta\sin(2pit) + \psi\sin(2pit)$, $F_\theta = \phi\cos(pit) + \psi\sin(pit)$ and $F_\varphi = \phi\cos(pit) + \theta\sin(pit)$. The time history of these disturbances is shown in Figs. 3 to 8. The control design parameters are chosen as $\lambda_L = \text{diag}(5,5,5)$, $k_L = \text{diag}(15,15,15)$, $K_c = \text{diag}(10,10,10)$, $\Gamma_1 = 1$, $\lambda_A = \text{diag}(10,10,10)$, $k_A = \text{diag}(45,45,45)$ and $\Gamma_2 = 1$. It is assumed that the state of the attitude dynamics are faster than the position dynamics. The tested results are given in Figs. 9 to 15. In view of these results, we can see that the position and attitude of the flying vehicle converges to the reference position and attitude even in the presence of modeling error and disturbance uncertainty.
Most importantly, the design does not require a priori known bound of the uncertainty. Evaluation results on custom made quadrotor UAV system has been given to demonstrate the theoretical development of this paper. Our evaluation showed that the proposed design can ensure the stability and tracking control property of the whole closed loop system for the given bounded uncertainty associated with modeling error and external disturbance.

IV. CONCLUSION

In this paper, we have introduced very simple adaptive sliding mode control system for quadrotor UAV system in the presence of uncertainty. Algorithms have developed by using Lyapunov function provided that all the states are available for measurement. The design can be used to compensate uncertainty associated with the modeling errors and external disturbances. Compared with other existing nonlinear adaptive backstepping control algorithms, the proposed design is simple and easy to implement as it does not require augmented variable and multiple design steps. Most importantly, the design does not require a priori known bound of the uncertainty. Evaluation results on custom made quadrotor UAV system has been given to demonstrate the theoretical development of this paper. Our evaluation showed that the proposed design can ensure the stability and tracking control property of the whole closed loop system for the given bounded uncertainty associated with modeling error and external disturbance.

REFERENCES


