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Adaptive Sliding Mode Control Design for Quadrotor Unmanned Aerial Vehicle

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***Abstract*— This paper addresses the stability and tracking control problem of quadrotor unmanned flying vehicle (UAV) in the presence of uncertainty. Adaptive autonomous sliding mode tracking system is designed by combining robust and adaptive control theory. Lyapunov analysis shows that the proposed algorithms can guarantee asymptotic convergence of the tracking error of the linear and angular motion dynamics of the vehicle. Compared with other existing adaptive backstepping design, the proposed method is very simple and easy to implement on an actual system as it does not require multiple design steps without augmented signals and *a priori* known bound of the uncertainty. To illustrate the theoretical argument, evaluation results on custom made quadrotor UAV system are presented for real-time applications.**

***Index Terms*— Quadrotor Aerial Vehicle; Lyapunov Method; Adaptive Control.**

I. Introduction

Recently, more and more researchers and industrial companies have focused their attention on designing a new type of small scale unmanned aerial vehicles. The interest in such small scale vehicles is growing in military and civilian applications, such as surveillance, inspection, search and rescue missions in dangerous or hostile environment. The design of autonomous flight control system for small scale quadrotor UAV in the presence of uncertainty is very difficult tasks due to its inherent nonlinearity associated with the dynamical model, underactuated property and external disturbances associated with uncertain flying environment. Over the past years, various automatic flight control systems for quadrotor system have been reported in the literature [2-6, 8-20]. Among these designs, PID and LQR type classical control mechanism has been widely used for commercial quadrotor system [1], [4], [5], [8], [10], [11], [19]. These classical algorithms may exhibit poor hovering performance because of the modeling error uncertainty. Backstepping control technique has been proposed to address the problem associated with the modeling error dynamics of the vehicle in [12, 15, 16, 17, 18]. Later, in [13], author included integral action with the backstepping technique in order to minimize the steady state tracking error. Most quadrotor unmanned flying robots are usually very small and lightweight, making the system sensitive to the variation in payload and uncertainty. This means that additional payload mass, moment of inertia, aerodynamic and gyroscopic force may change vehicles dynamic, affecting the stability and tracking performance significantly. Furthermore, unpredictable changes in flying environment may increase the modeling error and uncertainty, making the flight control system design even more complicated. As a result, available classical flight control system may not be able to deal with the change in flight dynamics for different flight missions. Therefore, the problem associated with uncertainty remains a challenging task demanding advanced autonomous flight control design for quadrotor UAV system. To deal with above mentioned problem, nonlinear control technique has been employed for designing autonomous quadrotor UAV system in [2], [3], [6], [9], [20]. In [2], authors have proposed so called dynamic inversion mechanism for hovering flight control system design by using well-known feedback linearization technique. H_∞ control technique combined with backstepping control mechanism in [6]. Nonlinear adaptive control algorithms using the backstepping technique proposed in [9]. Most recently, adaptive backsteeping control algorithm technique used to design nonlinear control for quadrotor UAV systems [3, 21]. However, the design and implementation mechanism of existing nonlinear control algorithms are very complicated as they usually associated with augmented auxiliary signals requiring multiple design and computation steps. So, our aim in this work is to develop very simple nonlinear adaptive flight control strategy which can cope modeling error and disturbances uncertainty. In this paper, we propose adaptive sliding mode control for stability and tracking control of quadrotor vehicle in the presence of uncertainty. Virtual adaptive position control algorithm combines gravity compensation with the desired linear acceleration and proportional-derivative error like terms. Adaptive attitude controller comprises proportional-derivative error like term with the desired angular acceleration term. Adaptation laws are used in both position and attitude dynamics to learn and compensate uncertainty associated with the variation of the payload mass, inertia, aerodynamic and gyroscopic force, external disturbances and unpredictable change in outdoor flying environment. Lyapunov method is employed to develop control algorithm and to analyze the convergence property of the linear and angular state dynamics. Unlike existing methods, the design does not use augmented signals and multiple steps which makes the design very simple and easy to implement for practical applications. Moreover, the design does not require *a priori* known bound of the uncer-

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tainty. Finally, various evaluation results on our laboratory made miniature quadrotor aerial vehicles are presented to demonstrate the effectiveness of the proposed method for practical applications.

II. Model Dynamics, Algorithm Design and Stability Analysis

We first derive mathematical model of the quadrotor flying vehicle [12, 13]. The position of the vehicle is defined as $P(t) = [x(t)\ y(t)\ z(t)]^T$ and its attitude represented by three Euler angles as $\Theta(t) = [\phi(t)\ \theta(t)\ \varphi(t)]^T$. The three translational and rotational velocities are defined as $V(t) = [V_1(t)\ V_2(t)\ V_3(t)]^T$ and $\Omega(t) = [\Omega_1(t)\ \Omega_2(t)\ \Omega_3(t)]^T$, respectively. Then, the relationship between velocities $(\dot{P}, \dot{\Theta}(t))$ and (V, Ω) can be written for earth fixed inertial reference frame and body fixed frame attached to the vehicle as follows

$$\dot{P} \;=\; \mathcal{R}_t(\Theta)V, \Omega = \mathcal{B}(\Theta)\dot{\Theta} \tag{1}$$

where $\mathcal{R}_t \in \Re^{3\times3}$ and $\mathcal{B} \in \Re^{3\times3}$ are the transformation velocity matrix and the rotation velocity matrix between fixed inertial frame and body fixed frame as given as follows

$$\mathcal{R}_t = \begin{bmatrix} C_\phi C_\varphi & S_\phi S_\theta C_\varphi - C_\phi S_\varphi & C_\phi S_\theta C_\varphi + S_\phi S_\varphi \\ C_\theta S_\varphi & S_\phi S_\theta S_\varphi + C_\phi C_\varphi & C_\phi S_\theta C_\varphi - S_\phi S_\varphi \\ -S_\phi & S_\phi C_\theta & C_\phi C_\theta \end{bmatrix} \tag{2}$$

$$\mathcal{B} = \begin{bmatrix} 1 & 0 & -S_\theta \\ 0 & C_\phi & C_\theta S_\phi \\ 0 & -S_\phi & C_\phi C_\theta \end{bmatrix} \tag{3}$$

where $S_{(.)}$ and $C_{(.)}$ denote $\sin(.)$ and $\cos(.)$, respectively. We now take the derivative equation (1) to constitute the kinematic equations for the quadrotor vehicle

$$\ddot{P} \;=\; \mathcal{R}_t\dot{V} + \dot{\mathcal{R}}_t V \tag{4}$$

$$\dot{\Omega} \;=\; \mathcal{B}\ddot{\Theta} + \left(\frac{\partial\mathcal{B}}{\partial\phi}\dot{\phi} + \frac{\partial\mathcal{B}}{\partial\theta}\dot{\theta}\right)\dot{\Theta} \tag{5}$$

Using $\dot{\mathcal{R}}_t = \mathcal{R}_t S(\Omega)$ with the skew-symmetric matrix $S(\Omega)$

$$S(\Omega) = \begin{bmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{bmatrix} \tag{6}$$

we can write equation (4) and (5) in the following form

$$\ddot{P} \;=\; \mathcal{R}_t\left(\dot{V} + \Omega \times V\right) \tag{7}$$

$$\dot{\Omega} \;=\; \mathcal{B}\ddot{\Theta} + \mathcal{D}(\Theta, \dot{\Theta}) \tag{8}$$

with

$$\mathcal{D}(\Theta, \dot{\Theta}) = \begin{bmatrix} -C_\theta\dot{\theta}\dot{\varphi} \\ -S_\phi\dot{\phi}\dot{\theta} + C_\phi C_\theta\dot{\phi}\dot{\varphi} - S_\phi S_\theta\dot{\theta}\dot{\varphi} \\ -C_\phi\dot{\phi}\dot{\theta} - S_\phi C_\theta\dot{\phi}\dot{\varphi} - C_\phi S_\theta\dot{\theta}\dot{\varphi} \end{bmatrix} \tag{9}$$

The dynamic equation of motion for the vehicle subjected to forces U_f and moments U applied to the center of the mass can be derived as

$$\ddot{P} \;=\; \beta U_f - \mathcal{C}\dot{P} - \gamma \tag{10}$$

$$\ddot{\Theta} \;=\; \mathcal{M}U - \eta\mathcal{D}(\Theta, \dot{\Theta}) - \xi\dot{\Theta} - \mathcal{B}\dot{\Theta} \times \mathcal{B}I\dot{\Theta} - \mathcal{B}\dot{\Theta} \times \Sigma_{i=1}^4 I_r\omega_i \tag{11}$$

Fig. 1. Custom Made Quadrotor UAV System.

where U_f is the force generated by the propellers, U is the total moments developed by the propellers, $\mathcal{M} = (I\mathcal{B})^{-1}$, $\eta = \mathcal{B}^{-1}$, $\beta = \left(m\mathcal{R}_t^T\right)^{-1}$ with constant payload mass m, $\mathcal{C} = m^{-1}L$ with aerodynamic drag coefficients $L = diag\,[L_{d1},\ L_{d2},\ L_{d3}]$ with $L_{d1} > 0$, $L_{d2} > 0$ and $L_{d3} > 0$, $\gamma = TH$ with $T = [0, 0, 1]^T$, $H = [0\ 0\ g\ ]^T$ and $g = 9.81\frac{m}{s^2}$, I_r is the inertia of the rotor blade, ω_i are the angular rotational velocities of the rotors and $\xi = I^{-1}M$ with symmetric positive definite constant inertia matrix $I = diag[I_x, I_y, I_z]$ and aerodynamic coefficients $M = diag\,[M_1,\ M_2,\ M_3]$ $M_1 > 0$, $M_2 > 0$ and $M_3 > 0$. Let us now introduce adaptive flight control strategy for the quadrotor UAV system given by equation (10) and (11). It is assumed that the translational and rotational dynamics are affected by external disturbance uncertainties as $F_a(t) = [F_x(t), F_y(t), F_z(t)]^T$ and $F_b(t) = [F_\phi(t), F_\theta(t), F_\psi(t)]^T$. We also assume that the desired task x_{1d}, x_{3d} and their first and second derivatives are bounded and belongs to a known compact set. Throughout our stability analysis, the position, orientation and their first derivatives are assumed to be available for measurement. Since $-\frac{\pi}{2} < \phi < \frac{\pi}{2}$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ and $-\pi < \psi < \pi$, the matrices $\mathcal{R}_t$ and $\mathcal{B}$ are bounded as $\|\mathcal{R}_t\| \le k_r$ with $k_r > 0$ and $\|\mathcal{B}\| \le k_t$ with $k_t > 0$. We now design adaptive flight controller for the attitude, altitude and virtual position dynamics such that (ϕ, θ, ψ) and (x, y, z) converges to the desired values of $(\phi_d, \theta_d, \psi_d)$ and (x_d, y_d, z_d). To do that, let us define the following state variables for the position and attitude dynamics as $x_1 = P$ and $x_3 = \Theta$. Then, the error model can be presented by the following state space equation

$$\begin{aligned} \dot{e}_1 &= e_2, \dot{e}_2 = -\beta U_f + \gamma + \mathcal{C}x_2 - F_a + \ddot{x}_{1d} \\ \dot{e}_3 &= e_4, \dot{e}_4 = -\mathcal{M}U - \zeta + \ddot{x}_{2d} \end{aligned} \tag{12}$$

where $U_f = \mathcal{U}_t$, $\zeta = f(x_3, x_4) + F_b$, $f(x_3, x_4) = -\eta\mathcal{D}(x_3, x_4) - \xi x_4 - \mathcal{B}x_4 \times \mathcal{B}Ix_4 - \mathcal{B}x_4\Sigma_{i=1}^4 I_r\omega_i$, $e_1 = (x_{1d} - x_1)$, $e_2 = (\dot{x}_{1d} - \dot{x}_1)$, $e_3 = (x_{2d} - x_3)$ and $e_4 = (\dot{x}_{2d} - \dot{x}_3)$. The, we define sliding surface for linear and angular dynamics as $S_L = (e_2 + \lambda_L e_1)$ and $S_A = (e_4 + \lambda_A e_3)$ with positive definite constant diagonal matrices $\lambda_L \in \Re^{3\times3}$ and $\lambda_A \in \Re^{3\times3}$. Then, we introduce the following adaptive control algorithm for $\mathcal{U}_t$

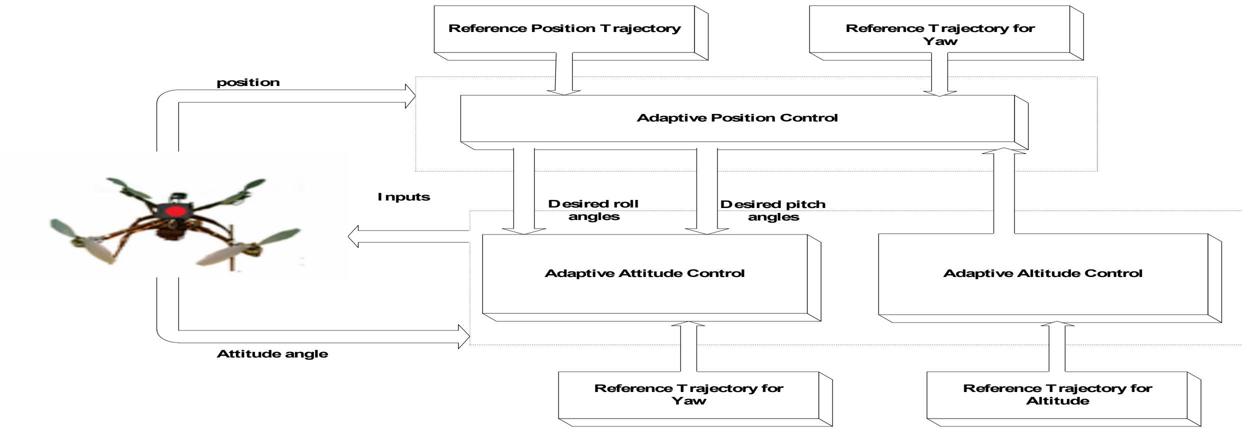

Fig. 2. Adaptive Autonomous Sliding Mode Tracking System for Quadrotor UAV System.

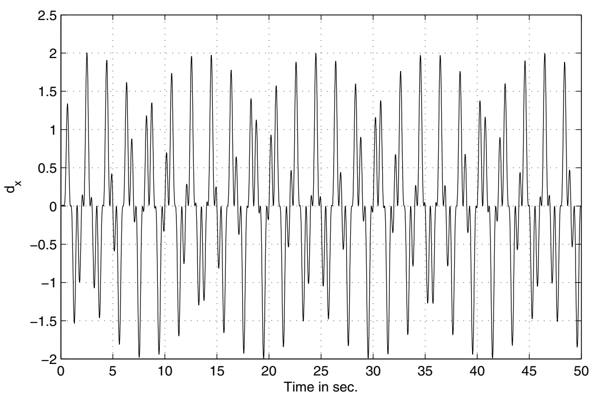

Fig. 3. Time history of d_x with x direction in meters.

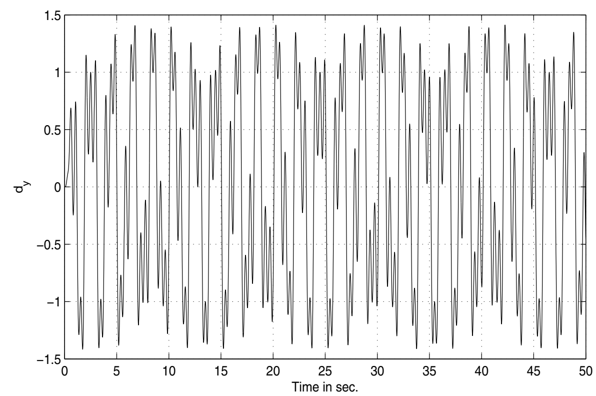

Fig. 4. Time history of state dependent disturbance d_y along with y direction in meters.

$$
\begin{aligned}
\mathcal{U}_t &= \beta^{-1}\left(\ddot{x}_{1d} + K_c x_2 + \lambda_L e_2 + \gamma + k_L S_L - u_l\right) \\
u_l &= \hat{\theta}_1 sign(S_L), \dot{\hat{\theta}}_1 = \Gamma_1 S_L^T sign(S_L) \qquad (13)
\end{aligned}
$$

where $\tilde{\theta}_1 = (\theta_1 - \hat{\theta}_1)$, $\hat{\theta}_1$ is the estimate of $\|F_a\| \leq \theta_1$, $K_c = diag[K_{c1}, K_{c2}, K_{c3}]$, $\lambda_L = diag[\lambda_{L1}, \lambda_{L2}, \lambda_{L3}]$, $k_L = [k_{L1}, k_{L2}, k_{L3}]$, $\Gamma_1 > 0$ and $\mathcal{U}_t = [\mathcal{U}_{tx}, \mathcal{U}_{ty}, \mathcal{U}_1]$. We now introduce the following adaptive control algorithm for $\mathcal{U}$

$$
\begin{aligned}
\mathcal{U} &= \mathcal{M}^{-1}\left(\ddot{x}_{2d} + \lambda_A e_4 + k_A S_A - u_A\right) \\
u_A &= \hat{\theta}_2 sign(S_A), \dot{\hat{\theta}}_2 = \Gamma_2 S_A^T sign(S_A) \qquad (14)
\end{aligned}
$$

where $\tilde{\theta}_2 = (\theta_2 - \hat{\theta}_2)$, $\hat{\theta}_2$ is the estimate of $\|\gamma_d\| \leq \theta_2$, $\lambda_A = diag[\lambda_{A1}, \lambda_{A2}, \lambda_{A3}]$, $k_A = [k_{A1}, k_{A2}, k_{A3}]$, $\Gamma_2 > 0$ and $\mathcal{U} = [\mathcal{U}_2, \mathcal{U}_3, \mathcal{U}_4]$. To avoid the discontinuity of the learning estimate, one needs to employ projection based learning estimate [22] as designed

$$
\begin{aligned}
\dot{\hat{\theta}}_1 &= Proj\left(\theta_1, \Gamma_1 S_L^T sign(S_L)\right) \qquad (15) \\
\dot{\hat{\theta}}_2 &= Proj\left(\theta_2, \Gamma_2 S_A^T sign(S_A)\right) \qquad (16)
\end{aligned}
$$

The desired roll, ϕ_d, and pitch angle, θ_d, in virtual adaptive position control can be calculated from the relationship $\phi_d = arcsin\left(\mathcal{U}_{tx}\sin(\psi_d) - \mathcal{U}_{ty}\cos(\psi_d)\right)$ and $\theta_d = arcsin\left(\frac{\mathcal{U}_{tx}\cos(\psi_d) + \mathcal{U}_{ty}\sin(\psi_d)}{\cos(\psi_d)}\right)$ with the given desired trajectory for yaw ψ_d. For algorithm design, stability and tracking convergence analysis, we consider the following composite Lyapunov functional

$$
V = \frac{1}{2}S_L^T S_L + \tilde{\theta}_1^T \Gamma_1^{-1} \tilde{\theta}_1^T + \frac{1}{2}S_A^T S_A + \tilde{\theta}_2^T \Gamma_2^{-1} \tilde{\theta}_2^T \qquad (17)
$$

We take the derivative (17) along the trajectories of the closed system formulated by using equation (12), (13), (14), (15) and (16). Then, $\dot{V}$ can be simplified as

$$
\dot{V} \leq -\lambda_{min}(k_L)\|S_L\|^2 - \lambda_{min}(k_A)\|S_A\|^2 < 0 \qquad (18)
$$

with $V \in \mathcal{L}_2$ and $V \in \mathcal{L}_\infty$. Then, in view of equation (18) and Barbalat's Lemma [14] together with projection mechanism (15) and (16), we can conclude that the error signals S_L and S_A are bounded and their bounds asymptotically converges to zero in Lyapunov sense. Since the signals S_L and S_A are bounded, then signals e_1, e_2, e_3 and e_4 are also bounded and their bounds also asymptotically converges to zero in Lyapunov sense.

III. Design Synthesis and Evaluation Results

To validate the control algorithm developed in previous section, various evaluations have been performed on

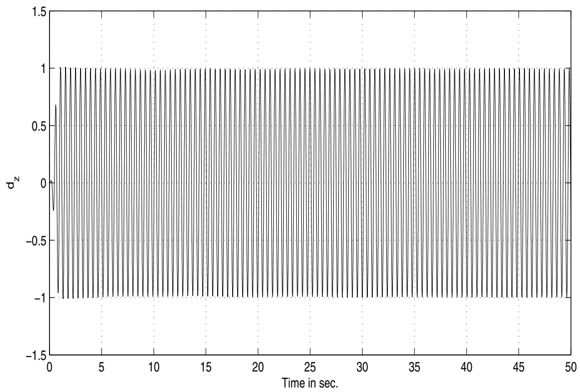

Fig. 5. Time history of state dependent disturbance d_z in z direction in meters.

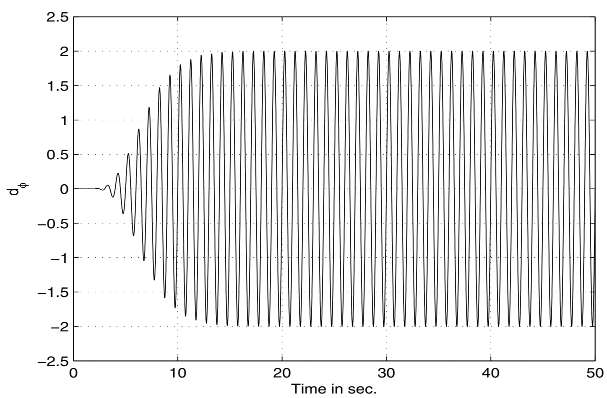

Fig. 6. External state dependent disturbance d_ϕ with ϕ direction in radians.

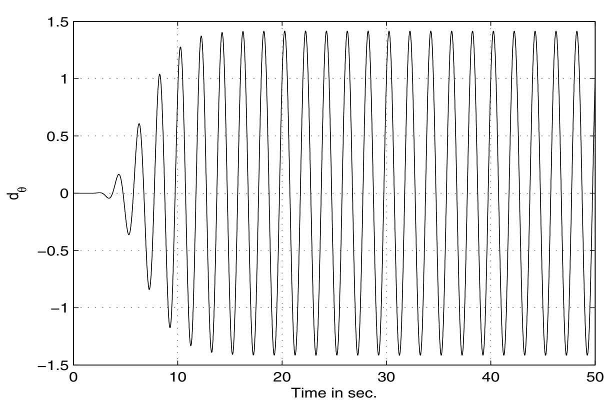

Fig. 7. Time history of d_θ in θ direction in radians.

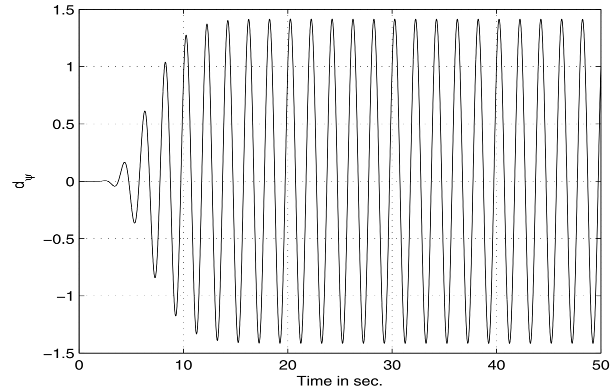

Fig. 8. Disturbance d_ψ with ψ direction in radians.

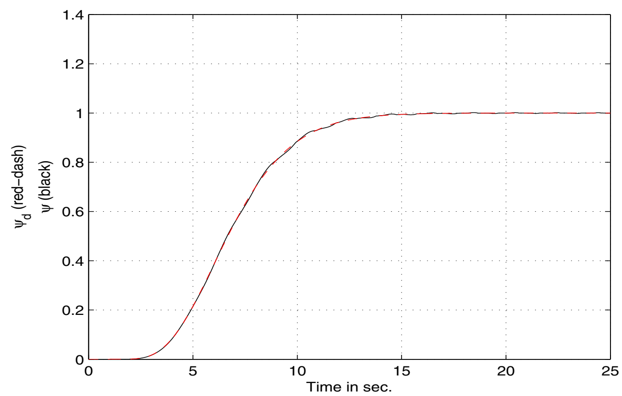

Fig. 9. Desired yaw angle task ψ_d (red-solid line) and actual output ψ (black-dash line) in radians.

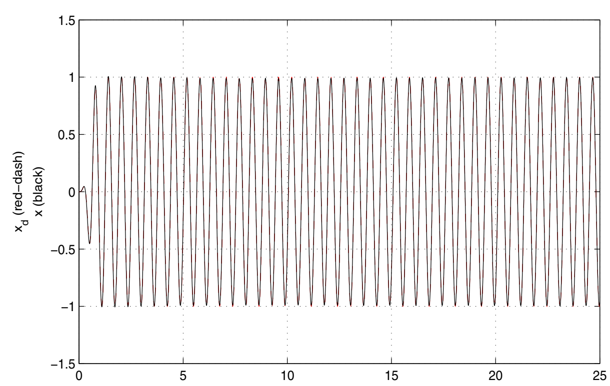

Fig. 10. Desired position tracking x_d (red-solid line) and actual output x (black-dash line) in meters.

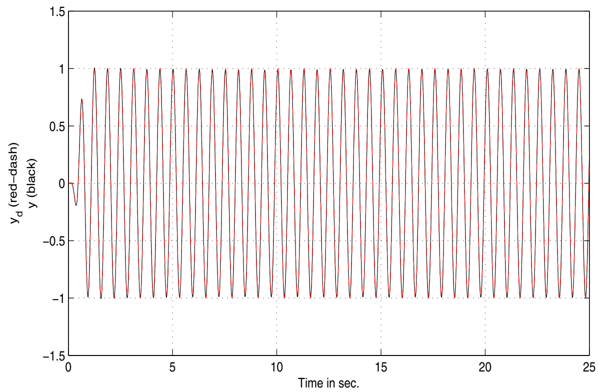

Fig. 11. Desired position tracking y_d (red-solid line) and actual output y (black-dash line) in meters.

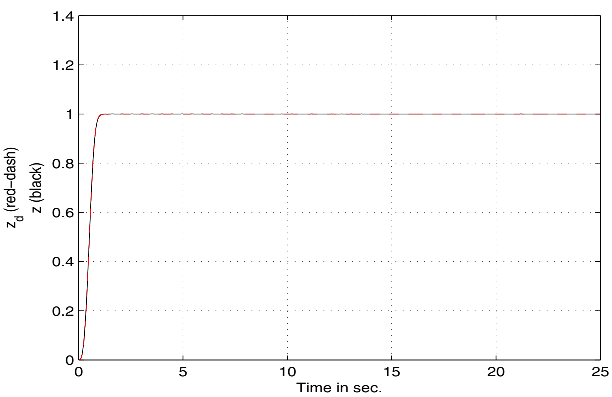

Fig. 12. Desired altitude tracking z_d (red-solid line) and actual output z (black-dash line) in meters.

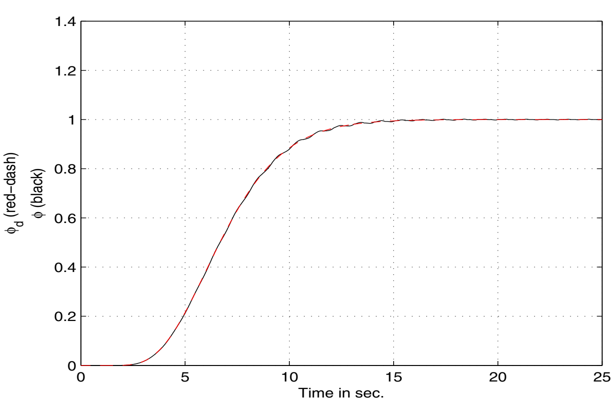

Fig. 13. Desired rolling angle ϕ_d (red-solid line) and actual output ϕ (black-dash line) in radians.

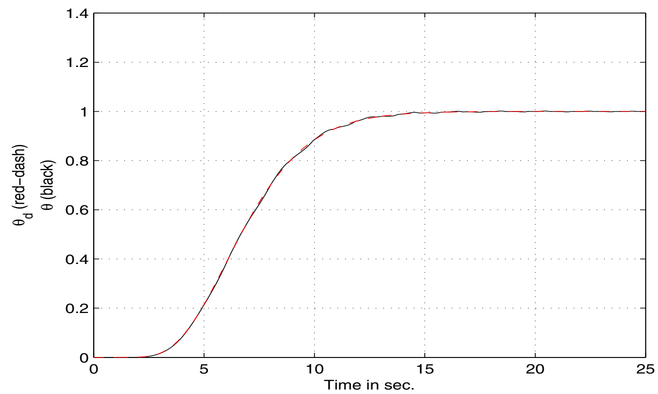

Fig. 14. Desired pitch angle θ_d (red-solid line) and actual output θ (black-dash line) in radians.

our laboratory made quadrotor UAV system as shown in Fig. 1. These evaluation results are based on the dynamic model presented by equation (10) and (11). In our evaluation, the desired trajectories for x_d, y_d and z_d are chosen as $x_d(t) = \left(1 - e^{-5t^3}\right) sin(10t)m$, $y_d(t) = \left(1 - e^{-5t^3}\right) cos(10t)m$, $z_d(t) = \left(1 - e^{-5t^3}\right) m$. The reference attitude angles ψ_d are generated by using the following transfer function $G(s) = \frac{1}{(s+1)^6}$. The desired roll, ϕ_d, and pitch angle, θ_d, is generated from the relationship $\phi_d = arcsin\left(\mathcal{U}_{tx}\sin(\psi_d) - \mathcal{U}_{ty}\cos(\psi_d)\right)$ and $\theta_d = arcsin\left(\frac{\mathcal{U}_{tx}\cos(\psi_d)+\mathcal{U}_{ty}\sin(\psi_d)}{\cos(\psi_d)}\right)$. Implementation diagram of the proposed adaptive sliding mode control algorithm is depicted in Fig. 2. The parameters of the the vehicle are selected as payload mass $m = 5\ kg$, distance from the center of the mass to the rotor axes $d = 0.2\ m$, lift constant $\alpha = 3.5 \times 10^{-5}\ N.\frac{s^2}{rad^2}$, $g = 9.81\ m.s^{-2}$, the drag factor $\alpha_d = .0032\ Nm.\frac{s^2}{rad^2}$, $I_x = 2\ Nm.\frac{s^2}{rad}$, $I_y = 3\ Nm.\frac{s^2}{rad}$, $I_z = 5\ Nm.\frac{s^2}{rad}$, $L_{d1} = 2\ N.\frac{s}{m}$, $K_{d2} = 5\ N.\frac{s}{m}$, $L_{d3} = 6\ N.\frac{s}{m}$, $M_1 = 3\ Nm.\frac{s}{rad}$, $M_2 = 5\ Nm.\frac{s}{rad}$ and $M_3 = 3\ Nm.\frac{s}{rad}$. To analyze the robustness of the proposed design, we choose the mass and inertial parameters four times larger than the actual values of the vehicle making large modeling error uncertainty. For our evaluation, the state dependant disturbances for the translational and rotational dynamics are chosen as $F_x = ysin(pit) + zsin(pit)$, $F_y = zsin(pit) + xcos(pit)$, $F_z = ysin(pit) + xcos(pit)$, $F_\phi = \theta sin(2pit) + \psi sin(2pit)$, $F_\theta = \phi cos(pit) + \psi sin(pit)$ and $F_\varphi = \phi cos(pit) + \theta sin(pit)$. The time history of these disturbances is shown in Figs. 3 to 8. The control design parameters are chosen as $\lambda_L = diag(5, 5, 5)$, $k_L = diag(15, 15, 15)$, $K_c = diag(10, 10, 10)$, $\Gamma_1 = 1$, $\lambda_A = diag(10, 10, 10)$, $k_A = diag(45, 45, 45)$ and $\Gamma_2 = 1$.It is assumed that the state of the attitude dynamics are faster than the position dynamics. The tested results are given in Figs. 9 to. 15. In view of these results, we can see that the position and attitude of the flying vehicle converges to the reference position and attitude even in the presence of modeling error and disturbance uncertainty.

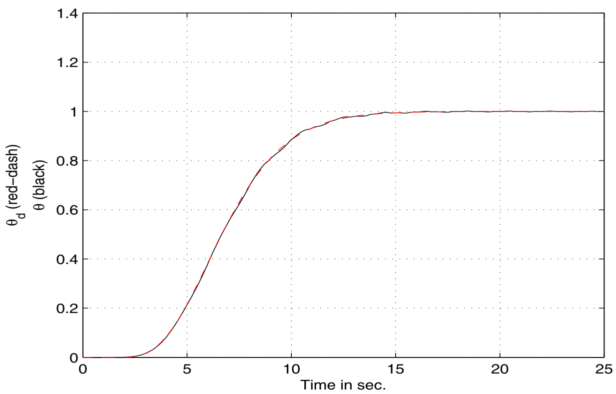

Fig. 15. Desired pitch angle θ_d (red-solid line) and actual output θ (black-dash line) in radians.

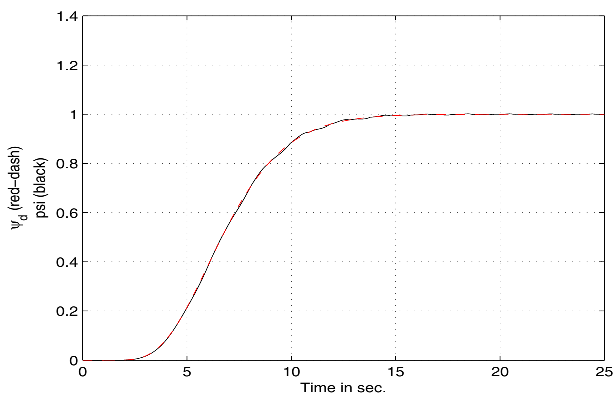

Fig. 16. Desired yaw angle task ψ_d (red-solid line) and actual output ψ (black-dash line) in radians.

IV. Conclusion

In this paper, we have introduced very simple adaptive sliding mode control system for quadrotor UAV system in the presence of uncertainty. Algorithms have developed by using Lyapunov function provided that all the states are available for measurement. The design can be used to compensate uncertainty associated with the modeling errors and external disturbances. Compared with other existing nonlinear adaptive backstepping control algorithms, the proposed design is simple and easy to implement as it does not require augmented variable and multiple design steps. Most importantly, the design does not require *a priori* known bound of the uncertainty. Evaluation results on custom made quadrotor UAV system has been given to demonstrate the theoretical development of this paper. Our evaluation showed that the proposed design can ensure the stability and tracking control property of the whole closed loop system for the given bounded uncertainty associated with modeling error and external disturbance.

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