KINEMATIC AND DYNAMIC MODELING OF A SIX LEGGED ROBOT

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Abstract: Legged vehicles can walk on rough and irregular surfaces with a high degree of softness . This is one of the main reasons why legged machines have received increasing attention by the scientific community. This article presents a simulator for a six leg machine. Both kinematic and dynamic models are developed. Kinematic equations are derived with Denavit-Hartenberg method. The Free Body Diagram method, based on the dynamic equations of isolated rigid bodies, is used to overcome the difficulties in dynamic modeling. Some results of simulation are presented.

Keywords: Legged robots, Hexapodal robots, Modelization, Dynamics, Kinematics

1. INTRODUCTION

Most of the vehicles that we are familiar with use wheels for their locomotion. Wheeled vehicles can achieve high speed motion with a relative low control complexity. Unfortunately they present several limitations in rough and irregular surfaces. Even with complex suspension systems they are only able to overcome relatively small irregularities on the terrain. The US army estimates that the wheel can only reach 50% of the places on earth. Whenever environment is a concern, the destruction maded in building suitable tracks is another problem ((Raibert, 1989)(Waldron, 1989*b*)(Waldron, 1989*a*)). The legged locomotion is one alternative that overcomes these difficulties. It introduces more flexibility and soil adaptation at the cost of lower speed and increased control complexity. Legged vehicles can walk on rough and irregular surfaces with a minimum of destruction and a high degree of softness. This explains the importance of legged robots on mobile robotics research.

The legged locomotion on natural terrain presents a set of complex problems (foot placement, obstacle avoidance, load distribution by the supports, general vehicle stability, etc) that must be taken into account both in mechanical construction of vehicles and in development of control strategies. One way to handle these issues is using models that mathematically describe the different situations. Therefore modelization becomes a useful tool in understanding systems complexity and in testing and simulating different control approaches.

Modelization techniques for legged mechanical structures are developed in this paper. The

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[2] This work was partially financed by NATO Scientific Affairs Decision on the framework of the NATO Science for Stability Program, PO-ROBOT project

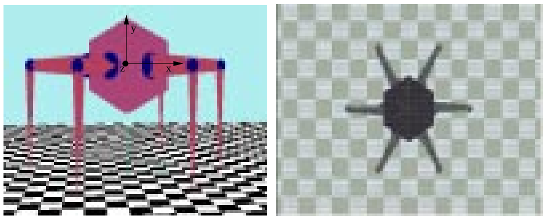

Fig. 1. The hexapodal structure

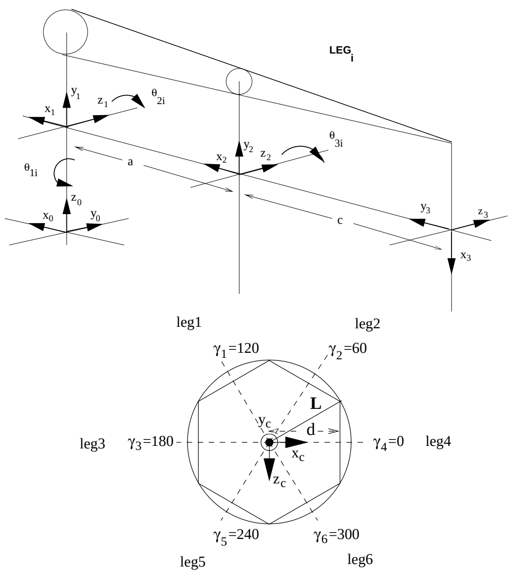

Fig. 2. The coordinate systems for each leg. Legs location around the central body.

Denavit-Hartenberg method is used in deriving a 3D kinematic model of a six leg robot. Dynamic modelization is performed using the Free Body Diagram method (FBD). The FBD method is introduced as an alternative to Lagrangian Formalism and is based in the dynamics of isolated rigid bodies. A simulator is built to validate achieved models. Some simulation results are presented at the end of the article.

2. THE KINEMATIC EQUATIONS

The considered 3D structure is formed by a central body, with an hexagonal shape and six legs. The legs are similar and simetrically distributed around the body (Fig.1). Each leg is composed by two links and three rotary joints (Fig.2). Two of these joints are located at the junction of the leg with central body (horizontal (θ_{1i}) and vertical (θ_{2i}) rotation). The third joint is located at the knee, connecting the upper and lower link (vertical rotation (θ_{3i})). Therefore each leg has 3 DOF (degree of freedom).Considering six legs and the additional 6 DOF for central body translation and rotation, the system has a total of 24 DOF.

2.1 *Direct kinematics*

$$ {}^{0}\boldsymbol{A_1} = \begin{bmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) & 0 \\ \sin(\theta_1) & 0 & -\cos(\theta_1) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1} $$

$$ {}^{1}\boldsymbol{A_2} = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & -a.\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & -a.\sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2} $$

$$ {}^{2}\boldsymbol{A_3} = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & c.\cos(\theta_3) \\ \sin(\theta_3) & \cos(\theta_3) & 0 & c.\sin(\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3} $$

$$ \vec{r_0} = \boldsymbol{F}(\theta_1, \theta_2, \theta_3) = {}^{0}\boldsymbol{A_1}(\theta_1).{}^{1}\boldsymbol{A_2}(\theta_2).{}^{2}\boldsymbol{A_3}(\theta_3).[0,0,0,1]^t \tag{4} $$

The Denavit-Hartenberg (D-H) method is one of the most popular technics used in kinematic modeling of manipulators (Fu, 1987)(Megahed, 1993)(Kanade, 1994). The described robot legs are similar to simple manipulators with 3 DOF. Therefore D-H method can be used to compute the transformation matrices between referential frames (Fig.2). Derived transformation matrices are presented in equations (1), (2), (3), where a and c are the length of links. Leg direct kinematic problem can be solved by these matrices. Function (4) shows it by computing tip coordinates on the base system given an arbitrary triad of joint angles $(\theta_1, \theta_2, \theta_3)$.

$$ {}^{c}\boldsymbol{A_{0i}} = \begin{bmatrix} -\cos(\gamma_i) & \sin(\gamma_i) & 0 & d.\cos(\gamma_i) \\ 0 & 0 & 1 & 0 \\ \sin(\gamma_i) & \cos(\gamma_i) & 0 & -d.\sin(\gamma_i) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5} $$

$$ {}^{i}\boldsymbol{A_c} = $$

$$ \begin{bmatrix} C(\phi_y)C(\phi_z) & S(\phi_y)S(\phi_x) - C(\phi_y)S(\phi_z)C(\phi_x) \\ S(\phi_z) & C(\phi_z)C(\phi_x) \\ -C(\phi_y)S(\phi_z) & S(\phi_y)S(\phi_z)C(\phi_x) + C(\phi_y)S(\phi_x) \\ 0 & 0 \end{bmatrix} $$

$$ \begin{matrix} C(\phi_y)S(\phi_z)S(\phi_x) + S(\phi_y)C(\phi_x) & R_x \\ -C(\phi_z)S(\phi_x) & R_y \\ C(\phi_y)C(\phi_x) - S(\phi_y)S(\phi_z)S(\phi_x) & R_z \\ 0 & 1 \end{matrix} \Bigg] \tag{6} $$

The six legs and the central body must be integrated to solve the global kinematic problem. Consider the referential located at the center of the body (Fig.2). Leg_i coordinates in body referential are obtained using the transformation matrix of equation (5). Note that $d = L.cos(\frac{\pi}{6})$ and each leg has a different γ_i associated with it. For simulation purposes it is important to be able to compute robot coordinates in an inertial frame located somewhere in space. The transformation matrix between body referential and the inertial frame is presented in equation (6). It depends of the 6 DOF of the robot central body. Three DOF are the angular positions (ϕ_x, ϕ_y, ϕ_z) of the body around the inertial axes. The other three are the coordinates of the mass center (R_x, R_y, R_z) in inertial frame. The considered rotation sequence is (Y,Z,X).

2.2 *Inverse kinematic*

$$(\theta_1, \theta_2, \theta_3) = \boldsymbol{F^{-1}}(p_x, p_y, p_z) \qquad (7)$$

Equation (4) computes tip coordinates given an arbitrary triad of joint angles values. The inverse function (equation 7) calculates joint angles given tip coordinatest. The derivation of inverse kinematics equations is essential for foot placement algorithms, trajectory planning, obstacle avoidance, etc.

$$F_{\theta_1}^{-1} = atan(\frac{p_y}{p_x}) \qquad (8)$$

$$F_{\theta_2}^{-1} = -acos(\frac{a^2 - c^2 + M^2}{2aM}) + +atan(-\frac{p_z}{\sqrt{p_x^2 + p_y^2}}) \qquad (9)$$

$$F_{\theta_3}^{-1} = \pi + acos(\frac{a^2 - c^2 + M^2}{2aM}) + +acos(\frac{c^2 - a^2 + M^2}{2cM}) \qquad (10)$$

$$M = \sqrt{p_x^2 + p_y^2 + p_z^2}$$

The function presented in equation (4) is injective, there is merely one inverse function. Using the transformation matrices derived above as well as some basic geometric concepts, equations (8), (9) and (10) are obtained.

$$\boldsymbol{\Omega} = \boldsymbol{J_{F^{-1}}}(p_x, p_y, p_z).\boldsymbol{V} \qquad (11)$$

Equation (11) relates the linear velocity of the tip with the angular velocity of the joints. This expression comes from the derivation of (7) where $\mathbf{J_{F^{-1}}}$ is the jacobian of $\mathbf{F^{-1}}$.

3. THE DYNAMIC EQUATIONS

Dynamic modeling of mechanical structures can be a complex problem. In robotics, more specifically, in manipulators, there are two main classical methodologies used for dynamic modeling: Lagrange and Newton-Euler.

The Lagrange approach is based on the energy principles. It works with scalar quantities, instead of vectors, handling the internal forces between the elements of the system in an implicit way. This method, although computationally expensive, can be particularly useful to obtain a state space model.(Fu, 1987) (Goldstein, 1950) (Wells, 1989).

The Newton-Euler method applies the vectorial dynamic equations to each element of the structure. The final system is achieved by joining all the elements equations. The internal forces are handled in an explicit way, as well as inertial and Coriolis forces. Most of the times Newton-Euler technique is difficult to use in modeling complex structures like legged robots.(Fu, 1987) (Megahed, 1993)

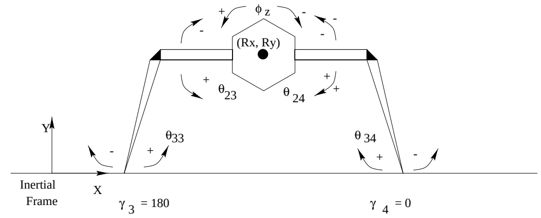

Fig. 3. Independent variables of 2D kinematic model.

In this work an alternative method, called Free Body Diagram (FBD), is used to model the dynamics of legged robots. It is based in dynamic equations of isolated rigid bodies (a standard in mechanics) and integrates some concepts of the last two methods.

The dynamic modeling of the 3D structure with six legs is a huge problem that would lead to a great amount of equations. Thus, to explain dynamic modeling using FBD approach, a simplified planar structure is considered. However the formalism of FBD method can be extended to 3D structures.

The simplified 2D structure has two legs and a central body (Fig.3). The legs used in the planar example are the 3 and 4 of the equivalent 3D structure (Fig.2). Each leg is composed by two links and two rotary joints (θ_{1i} disappears). Considering central body with 3DOF (translation in X and Y and rotation around Z), the system has a total of 7 DOF. It is necessary to reach a kinematic model of the system and to select the independent or generalized kinematic variables.

Planar transformation matrices $\mathbf{^2A_3}$, $\mathbf{^1A_2}$, ${}^cA_1$, ${}^iA_c$ are derived from 3D transformation matrices. Note that if values of - $R_x, R_y, \phi_z, \theta_{23}, \theta_{24}, \theta_{33}, \theta_{34}$ - are known at a given instant of time, structure position can be determined in an unambiguous way. These variables are the seven independent kinematic variables (or the independent generalized variables of Lagrange Formalism (Wells, 1989)).

4. INTRODUCTION TO THE FBD METHOD

$$\sum F_x^{ext} = M.a_x \qquad (12)$$

$$\sum F_y^{ext} = M.a_y \qquad (13)$$

$$\overrightarrow{\boldsymbol{M_z}} = \sum_i \overrightarrow{\boldsymbol{r_i}} \times \overrightarrow{\boldsymbol{F_i^{ext}}} \qquad (14)$$

$$M_z = I_{CM}.\gamma \qquad (15)$$

The FBD method is based in rigid body dynamics. Given a 2D body, its position in an inertial

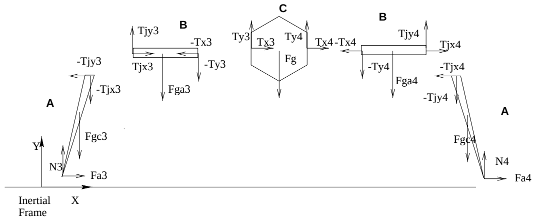

Fig. 4. Free Body Diagram

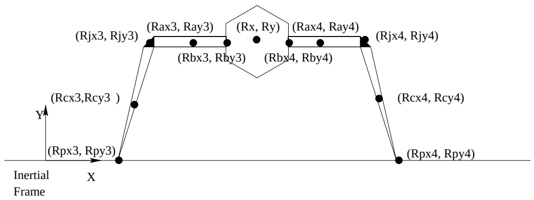

Fig. 5. Auxiliary points

frame is determined in an unique way by the XY coordinates of the center of mass (CM) and a rotation angle. Equations (12), (13), (14) and (15) describe the dynamic behavior of the body when a set of external forces F_i^{ext} is applied. Equations (12) and (13) calculate the translational motion of the CM due to the applied resultant force (in X and Y directions). Equation (14) computes the force moment M_z and equation (15) determines the angular acceleration. Notice that M is the mass and I_{CM} is the inertial moment for rotation around an axis parallel to inertial frame Z axis and passing through CM. If an inertial moment I_P is considered the rotation axis will pass through a given point P instead of CM.

This formulation can be extended to a 3D body. Its position is determined by XYZ coordinates of CM, thus it has three dynamic equations for translation. If the body motion is completely free and can rotate around inertial X, Y and Z directions, three rotation equations are needed. For the general case the dynamic behavior is described by six equations.

In the study example, 2D structure is formed by five rigid bodies: the central body (cp), the superior links (a3, a4) of both legs and the inferior links (c3, c4). Each element must be isolated to build the FBD diagram with all the applied forces (Fig.4). The external forces to the structure are the gravitic forces and the friction forces on the tips (friction on the joints is not considered). The contact forces between the different elements are internal to the system.

4.1 *Auxiliary points coordinates*

To avoid dealing with inertial forces and complex transformations, dynamic equations are derived in an inertial referential located outside the structure. Therefore the inertial coordinates of the points where forces are applied must be calculated . These points are: the mass centers (gravitical forces), the contact points of the links(tension forces) and the supports (friction forces)(Fig. 5). They are essential to compute the rotation force moments . The arm vectors coordinates (r_i in equation 14) can be determined by subtracting pairs of points coordinates in the inertial frame referential.

With planar transformation matrices the derivation of inertial points coordinates as a function of the seven independent kinematic variables is straight forward. These functions describe the restrictions in motion imposed by structure configuration. In the example of 2D structure the five rigid bodies are connected and, consequently, the motion of each one is dependent of the others.

4.2 *Rotation versors of isolated rigid bodies*

$$\overrightarrow{\boldsymbol{M^r}} = \overrightarrow{\boldsymbol{M}}.\overrightarrow{\boldsymbol{vers}} \tag{16}$$

For each isolated body, the resultant force moment ($\overrightarrow{M}(\mathbf{M_x}, \mathbf{M_y}, \mathbf{M_z})$) is calculated in an inertial frame placed outside the structure. To determine the angular acceleration it is necessary to compute the component of force moment ($\overrightarrow{M^r}(\mathbf{M^r_x}, \mathbf{M^r_y}, \mathbf{M^r_z})$) with the direction of body rotation axis (equation 15). In D-H method the Z axis of the referential frames attached to the each joint is coincident with the joint rotation axis (Fig 2). Thus, rotation axis versor $\overrightarrow{vers}$ is determined by computing Z versor coordinates in the inertial referential frame. These are easily derived using the kinematic transformation matrices. $\overrightarrow{M^r}$ is calculated in equation (16) using scalar product.

Considering the 2D structure, the rotation axis of the central body ($\overrightarrow{vers_{cp}}$) and both links of leg 3 ($\overrightarrow{vers_{a3}}, \overrightarrow{vers_{c3}}$) have the same direction and orientation of the Z axis in the inertial frame. For leg 4 ($\overrightarrow{vers_{a4}}, \overrightarrow{vers_{c4}}$) the direction is the same of Z, but the orientation is symmetric.

4.3 *The system of dynamic equations*

From the FBD diagram (Fig.4) the dynamic equations of each element of 2D structure can be derived. For the central body (C):

$$\begin{cases} m_{cp}\dfrac{d^2R_x}{dt^2} = T_{x3} + T_{x4} \\ m_{cp}\dfrac{d^2R_y}{dt^2} = T_{y3} + T_{y4} - m_{cp}g \\ \overrightarrow{\boldsymbol{M_{cp}}} = \displaystyle\sum_{i=3}^{4}[(\overrightarrow{\boldsymbol{R_{bi}}} - \overrightarrow{\boldsymbol{R}})\times\overrightarrow{\boldsymbol{T_{yi}}}] \\ I_{cp}\dfrac{d^2\phi_z}{dt^2} = \overrightarrow{\boldsymbol{vers_{cp}}}.\overrightarrow{\boldsymbol{M_{cp}}} \end{cases} \tag{17}$$

For the upper link of generic leg_i (B) (Fig.4):

$$\begin{cases} m_a \dfrac{d^2 R_{axi}}{dt^2} = T_{jxi} - T_{xi} \\ m_a \dfrac{d^2 R_{ayi}}{dt^2} = T_{jyi} - T_{yi} - m_a g \\ \overrightarrow{\boldsymbol{M_{ai}}} = [(\overrightarrow{\boldsymbol{R_{bi}}} - \overrightarrow{\boldsymbol{R_{ai}}}) \times (-\overrightarrow{\boldsymbol{T_{yi}}})] + \\ +[(\overrightarrow{\boldsymbol{R_{ji}}} - \overrightarrow{\boldsymbol{R_{ai}}}) \times \overrightarrow{\boldsymbol{T_{ji}}}] \\ \dfrac{d^2\theta_{2i}}{dt^2} = \dfrac{\overrightarrow{\boldsymbol{vers_i}} . \overrightarrow{\boldsymbol{M_{ai}}}}{I_a} - \dfrac{\overrightarrow{\boldsymbol{vers_{ai}}} . \overrightarrow{\boldsymbol{M_{cp}}}}{I_{cp}} \end{cases} \quad (18)$$

For the lower link of generic leg_i (A) (Fig.4):

$$\begin{cases} m_c \dfrac{d^2 R_{cxi}}{dt^2} = F_{ai} - T_{jxi} \\ m_c \dfrac{d^2 R_{cyi}}{dt^2} = N_i - T_{jyi} - m_c g \\ \overrightarrow{\boldsymbol{M_{ci}}} = [(\overrightarrow{\boldsymbol{R_{ji}}} - \overrightarrow{\boldsymbol{R_{ci}}}) \times (-\overrightarrow{\boldsymbol{T_{ji}}})] + \\ +[(\overrightarrow{\boldsymbol{R_{pi}}} - \overrightarrow{\boldsymbol{R_{ci}}}) \times (\overrightarrow{\boldsymbol{F_{fi}}})] \\ \dfrac{d^2\theta_{3i}}{dt^2} = \dfrac{\overrightarrow{\boldsymbol{vers_{ci}}} . \overrightarrow{\boldsymbol{M_{ci}}}}{I_c} - \dfrac{\overrightarrow{\boldsymbol{vers_{ci}}} . \overrightarrow{\boldsymbol{M_{ai}}}}{I_a} \end{cases} \quad (19)$$

The eight last equations are generic for leg_i (i=3,4).The inertial mass of the central body, the superior link and the inferior link are respectively m_{cp}, m_a and m_c. The inertial moments I_{cp}, I_a, I_c are computed for the CM of the constituents, g is the gravitic acceleration and $\overrightarrow{F_{fi}}(\mathbf{F_{ai}}, \mathbf{N_i})$ (i=3,4) refers to the friction force.

Notice that the last rotation equation of (18) and (19). Beside the use of versors explained above, diferential angular acceleration between two consecutive elements are computed, instead of absolute acceleration referred to the semi-positive X axis. This is done as a way to avoid more equations and variables. The body rotation angle is the same in the kinematic and dynamic modeling.

For 2D structure example 20 dynamic equations are obtained. In previous sections more 25 kinematic equations were derived to compute the auxiliary points $(\mathbf{R_{ai}}, \mathbf{R_{ci}}, \mathbf{R_{bi}}, \mathbf{R_{ji}}, \mathbf{R_{pi}})$ and the rotation versors $(\overrightarrow{vers_{cp}}, \overrightarrow{vers_{ai}}, \overrightarrow{vers_{ci}})$ in function of the seven independent kinematic variables. If applied external forces are known the dynamic description of the system is concluded.

4.4 *Foot-soil interaction*

In this example the external forces are the gravitic forces and the friction forces at the tips. The first ones are constant and easy to compute, but the same can't be said for the second ones.

There are a wide variety of foot soil interaction modeling. The model that is used considers a rigid surface whose shape is described in the inertial frame by $y = \varrho(x)$. The friction forces $\overrightarrow{F_f}(\mathbf{F_a}, \mathbf{N})$ (N is the normal reaction of the soil) are evaluated with the help of two coefficients, μ_S and μ_D, dependent of the soil features. If the tip doesn't slide along the surface than $F_a \leq \mu_S \times N$, else $F_a = \mu_D \times N$. Usually $\mu_S \geq \mu_D$. In each instant of time the leg must be in one of three states: non contact(ST3), contact without sliding(ST2), and contact with sliding(ST3).

ST1	**ST2**	**ST3**
$N_i = 0$	$R_{pxi} = a$	$F_{ai} = \mu_D . N_i$
$F_{ai} = 0$	$R_{pyi} = b$	$R_{pyi} = \varrho(R_{pxi})$

If the leg is in state ST1 then $R_{pyi} > \varrho(R_{pxi})$ and (R_{pxi}, R_{pyi}) are the tip coordinates. The leg is raised and no forces are exerted on the tip. The first column equations are added to the global system of equations .

When the leg is in contact with the soil at point (a,b) two cases can be considered. If it doesn't slip along the surface (state ST2) than friction force can't be directly computed. However tip remains in the same position whose coordinates are known. Second column equations are added to the global system and determination of (N_i, F_{ai}) becomes possible. If the computed value of F_{ai} is higher than $\mu_S . N_i$ it means that the static friction force is not enough to keep the tip in (a,b). In this case the leg is in state ST3. Third column equations, instead of second column, are added to the global system whose solutions are recalculated.

4.5 *Mathematical resolution*

Derived a system with the same number of variables and independent equations (non singular), solutions must be determined. Consider a period of time Δt and make the discrete sampling of the variables. A system of non-linear discrete equations is obtained by replacing the second order derivatives by a discrete approximation. In each moment $t = k.\Delta t$ system solution gives the applied forces $(\overrightarrow{T_i}, \overrightarrow{T_{ji}}, \mathbf{etc})$ and predicts the position of the structure in the next moment$(R_x[k+1], R_y[k+1], \phi_z[k+1], etc)$.

To simulate the 2D structure behavior in a continuous way the set of derived equations must be successively solved (k=0,1,2,...). Consider the instant of time $n.\Delta t$. Structure position was determined in last iteration (k=n-1). Therefore, using the 25 kinematic equations, the inertial coordinates of auxiliar points and rotation versors can be directly computed. Replacing them in expressions (17), (18) and (19) the dimension of non-linear algebraic system is reduced in the 25 kinematic equations. That is an important advantage of discrete sampling. The resultant system has 24 discrete dynamic equations that can be solved by applying the Newton method.

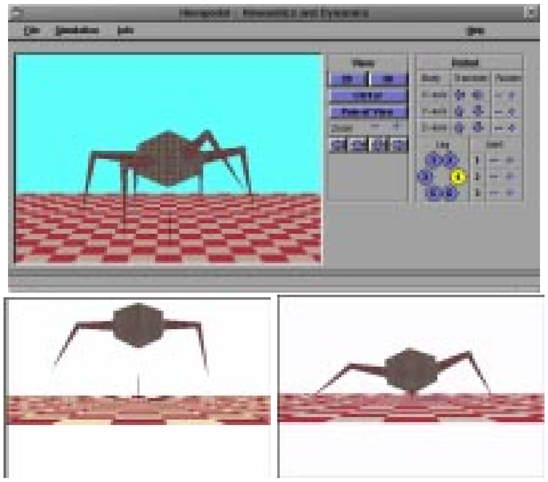

Fig. 6. The hexapodal simulator. Views of the structure during the fall

5. SIMULATION AND RESULTS

A kinematic and dynamic simulator was programmed using the derived models (Fig.6). In this section the results of one of the many realized experiments is presented.

R_x	R_y	ϕ_z	θ_{23}	θ_{33}	θ_{24}	θ_{34}
0	0.6	0	0	245	0	255

Consider that the structure falls from a starting position (Fig. 6). The initial values of independent kinematic variables are in the table. The considered period of time is 0.01s and the friction coefficients are $\mu_S = 0.4$ and $\mu_D = 0.3$. Note that for t=0 the structure is not in contact with the ground.

To illustrate the studies that can be realized using the simulator, Fig.7 depicts the evolution tip's position and applied friction forces. Note that leg 4 touches the ground 0.02s before leg 3. The first one reaches the soil 0.20s after the start of motion. Around moment 0.24s both legs start slipping.

6. DISCUSSION AND CONCLUSIONS

In this article we have exploited basic rigid body dynamic concepts to model complex structures. The FBD dynamic equations are intuitive, easy to derive and allow the computation of the internal forces and moments.

Lagrange Formalism is a powerful tool that can be particularly useful when a state space model of the dynamic system is intended. Some efforts have been done to reach a state space model of the described 2D structure using Lagrange approach. However, due to the complexity of the derived expressions, the symbolic inversion of matrix **D** ((Fu, 1987)(Manko, 1993)), when possible, needs a huge amount of computation time.

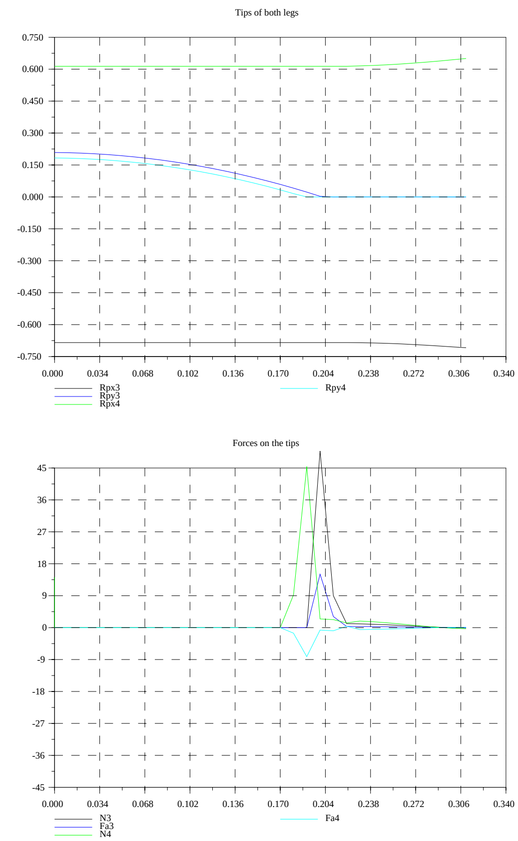

Fig. 7. Graph1: Evolution of tips position of both legs in inertial frame (Position(m) versus Time(s)). Graph2: Applied forces on the tips of both legs (Force(N) versus Time(s)).

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