Multi-Robot Complete Exploration using Hill Climbing and Topological Recovery

Rui Rocha†, Filipe Ferreira†‡ and Jorge Dias†

{*rprocha** | *cfferreira* | *jorge*}*@isr.uc.pt*

†Instituto de Sistemas e Robótica
Department of Electrical and Computer Engineering
University of Coimbra, 3030-290 Coimbra, Portugal

‡Department of Mechanical Engineering
University of Aveiro, 3810-193 Aveiro, Portugal

Abstract— This article addresses the problem of autonomous map building and exploration of an unknown environment with mobile robots. The proposed method assumes that mobile robots use occupancy grid maps as the main representation model for the built maps and a hill climbing local search algorithm for exploring the environment without any kind of human intervention. It is demonstrated that hill climbing based exploration may recover from local minima and cover completely any environment, if a topological representation of the environment is created incrementally along the mapping and exploration mission. The approach is devised for either a single mobile robot or multiple cooperative mobile robots.

Index Terms— Autonomous exploration, map building, multi-robot systems, occupancy grids, topological maps.

I. INTRODUCTION

One of the basic capabilities of autonomous mobile robots performing a mission on an unknown environment (*e.g.* surveillance, reconnaissance, tracking moving objects, collecting objects, *etc*.) is to acquire a representation model of it, which may be taken as a basis to localization, path planning, navigation and other mission-specific tasks. This is required for either indoor [1] or outdoor environments [2].

Simultaneous Localization and Mapping (SLAM) has been extensively studied for the past few years [3] with the aim of estimating the robot's trajectory and build at the same time a map of the environment. Since SLAM requires that the area of interest be completely covered by the robots' sensors, an important related problem is the action selection problem which has received surprisingly much less attention than SLAM, though it is also crucial for deploying autonomous mobile robots in unknown environments. The action selection problem can be stated informally as selecting exploration views which maximize the utility of new sensory information in every sensing cycle, so as to minimize the time needed to completely explore the environment. In the multi-robot case, mobile robots have also to coordinate their actions.

Yamauchi *et al.* proposed frontier-based exploration [4] whereby robots are driven towards boundaries between open space and unexplored regions. More recently, Burgard *et al.* used this concept to develop a technique for coordinating a team of robots when building 2-D occupancy grids [1], which uses the value iterated algorithm to formalize a balance between travel cost and utility of unexplored regions, so that robots simultaneously explore different regions. However, the technique is computationally prohibitive for large environments as it is a global search technique that requires the computation of value functions along the entire environment.

Rocha *et al.* developed a mapping framework based on grid maps and entropy [5], [6]. They formalized the Yamauchi's frontier-cell concept as an empty cell with high entropy gradient, and the frontier-based exploration as a hill climbing technique. This local search technique was also extended to support cooperative multi-robot exploration [7]. Although the method is computationally more efficient than the one in [1], it may not be able to cover entirely any environment because of local minima. The method is thus complemented in this article with the creation of a topological graph of the environment containing a roadmap of free space, which allows to recover hill climbing exploration in the presence of local minima. With residual extra computational effort, the method is virtually able to explore any environment, although it is still essentially a local search method, much more scalable than the approach in [1].

Baños *et al.* developed the safe region concept and the next best view (NBV) algorithm [8] within polygonal maps, in order to select candidate views that balance information gain against cost. The method presented herein is rooted on the same basic idea but it is formulated for occupancy grid maps instead of polygonal maps. Some motion planning methods, including Probabilistic Roadmaps (PRM) [9] and Rapidly-exploring Random Trees (RRT) [10], are based on the exploration of a topological graph of the state space. The work presented herein also uses this kind of topological information, but it is driven by an entropy-based measure rather than performing a random walk in the state space.

Section 2 summarizes entropy-gradient based exploration [5], [6] and states the local minima problem. Section 3 presents the topological recovery technique for a single mobile robot, which is extended to multiple robots in section 4. The article ends with conclusions and future work.

II. HILL CLIMBING EXPLORATION

This section summarizes the exploration algorithm proposed in [5], [6]. It was formalized in a framework that

uses volumetric grid maps to represent the 3-D environments. Although it is sufficiently generic to support 3-D information and information about cell's coverage[1], in this article conventional 2-D occupancy grid maps [1] are used for the sake of simplicity.

An occupancy grid map m discretizes the environment being mapped in an evenly spaced grid of cells $m_i \in m$ [1]. Building an occupancy grid map means to calculate the posterior over maps $p(m \mid z_{1:t}, x_{1:t})$, given the set of all measurements $z_{1:t}$ up to time t and the path $x_{1:t}$ of the robot. It is commonly assumed that occupancy of probabilities of different cells are statistically independent. Thus, being $p(m_i)$ the probability that a grid cell is occupied, the occupancy grid map is just

$$p(m \mid z_{1:t}, x_{1:t}) = \prod_i p(m_i \mid z_{1:t}, x_{1:t}). \tag{1}$$

Occupancy estimates are extracted from new sensory information using an inverse sensor model $p(m_i \mid z_t, x_t)$. For each measurement z_t, the set of cells m_i in its perceptual field is computed, using a ray-casting technique in the occupancy grid. New estimates may be integrated in the map using the log odds implementation of the Bayes filter described in [1], which estimates $p(m_i \mid z_{1:t}, x_{1:t})$, given $p(m_{i-1} \mid z_{1:t-1}, x_{1:t-1})$ and $p(m_i \mid z_t, x_t)$.

Entropy is a measure of the uncertainty of a probability distribution. The entropy of a cell is just

$$H(m_i) = -p(m_i) \log [p(m_i)] - (1 - p(m_i)) \log_2 [1 - p(m_i)], \tag{2}$$

being $H(m_i) = 0$, for $p(m_i) = 0$ or $p(m_i) = 1$, and $H(m_i) = 1$, for $p(m_i) = 0.5$, assuming base 2 logarithms.

The entropy value $H(m_i)$ is a sample of a continuous entropy field $H : \mathbb{R}^2 \to \mathbb{R}$, taken at cartesian position $\mathbf{w}_i \in \mathbb{R}^2$, the center of cell m_i. Let $m_i^{\Theta-}$ denote the contiguous cell to m_i in the negative direction of axis Θ, and ϵ the edge of a cell. A reasonable first order approximation to the entropy gradient at $\mathbf{w}_i$ is

$$\overrightarrow{\nabla} H(m_i) \approx \frac{1}{\epsilon} \left[H(m_i) - H(m_i^{x-}), \quad H(m_i) - H(m_i^{y-}) \right]^T, \tag{3}$$

having magnitude $\left\| \overrightarrow{\nabla} H(m_i) \right\|$ and direction given by the unitary vector

$$\hat{\mathbf{p}}(m_i) = \frac{\overrightarrow{\nabla} H(m_i)}{\left\| \overrightarrow{\nabla} H(m_i) \right\|}, \quad \overrightarrow{\nabla} H(m_i) \neq \overrightarrow{0}. \tag{4}$$

Let $\rho(\mathbf{x}, m_i) \in [0; 1]$ denote a coefficient which measures if a cell m_i is in line-of-sight from a position $\mathbf{x}$, which also implies that cell m_i is likely to be empty. This coefficient can be easily computed from current map $p(m \mid z_{1:t}, x_{1:t})$ using a ray-casting technique in the occupancy grid. Given the current robot's position x_t, any cell m_i is considered as a frontier cell if $\rho(x_t, m_i)$ is greater than a given threshold ρ_{th} and entropy gradient $H(m_i)$ is greater than a given entropy

[1] An extension of an occupancy grid map was used which is able to model cells that are only partially occupied.

threshold $H_{th} > 0$. This means that a frontier cell is an empty cell, in line-of-sight from current robot's position, which is contiguous to an unexplored cell m_j ($p(m_j) \approx 0.5$).

Let denote the applied vector connecting a cartesian position $x \in \mathbb{R}^2$ to the center of a cell m_i as $\overrightarrow{\mathbf{u}}(\mathbf{x}, m_i) = \mathbf{w}(m_i) - \mathbf{x}$. The exploration algorithm computes the set of frontier cells located in a neighborhood with radius ε

$$\mathcal{N}(x_t, \varepsilon) = \Big\{ m_i : \mathbf{u}(x_t, m_i) \| \leq \varepsilon, \ \rho(x_t, m_i) \geq \rho_{th},$$
$$\left\| \overrightarrow{\nabla} H(m_i) \right\| \geq H_{th} \Big\}, \tag{5}$$

whose centers $\mathbf{w}(m_i)$, $m_i \in \mathcal{N}(x_t, \varepsilon)$, are candidate exploration views. The best frontier cell is selected as

$$m_i^s = \underset{m_i \in \mathcal{N}(x_t, \varepsilon)}{\operatorname{argmax}} \left[\rho(x_t, m_i) \left\| \overrightarrow{\nabla} H(m_i) \right\| \right]. \tag{6}$$

Then the robot navigates straight towards the position $\mathbf{w}(m_i^s)$ with a gaze on arrival defined by the direction of $\hat{\mathbf{p}}(m_i^s)$.

The hill climbing technique summarized in equation (6) is successful if $\mathcal{N}(x_t, \varepsilon) \neq \emptyset$, otherwise there is no any frontier cell and the robot is stuck on a local minima of the map's entropy gradient. Fig. 1 shows an example of map building in a typical office environment: on the left, it shows the environment and the places visited by the robot, on the middle the occupancy grid map (black, white and grey regions mean, respectively, high occupancy, low occupancy and high entropy) and on the right the entropy field. The two rows in the figure depict two different situations of hill climbing exploration: on the top row, the robot selects successfully the best frontier cell in its neighborhood (top-right graph); conversely, the bottom row presents a situation wherein entropy is virtually null in the robot's neighborhood, corresponding to a local minima situation. A method is required to overcome this situation and re-start exploration on farther regions (bottom-right graph).

III. TOPOLOGICAL RECOVERY OF LOCAL SEARCH EXPLORATION WITH A SINGLE ROBOT

The situation depicted on the bottom of Fig. 1 demonstrates the need for a method to recover the local search hill climbing exploration algorithm from a local minima. There is only one particular case in which a local minima is a desirable situation: the end of the exploration. In this case, there is no any position x_t for which the set of cells given by equation (5) is not empty, therefore the local minima found by the mobile robot is *indeed* a global minima. In other cases, the local minima means that the robot did not explore completely every frontier cells along its exploration path, since the beginning of the process, and therefore should revisit previous places where there are still regions to be explored (see the bottom-middle map in Fig. 1).

A. Topological map

In order to find previous places that should be revisited, it is necessary to keep track of the environment topology during the exploration process. The solution is to estimate the occupancy grid map and, simultaneously, build incrementally
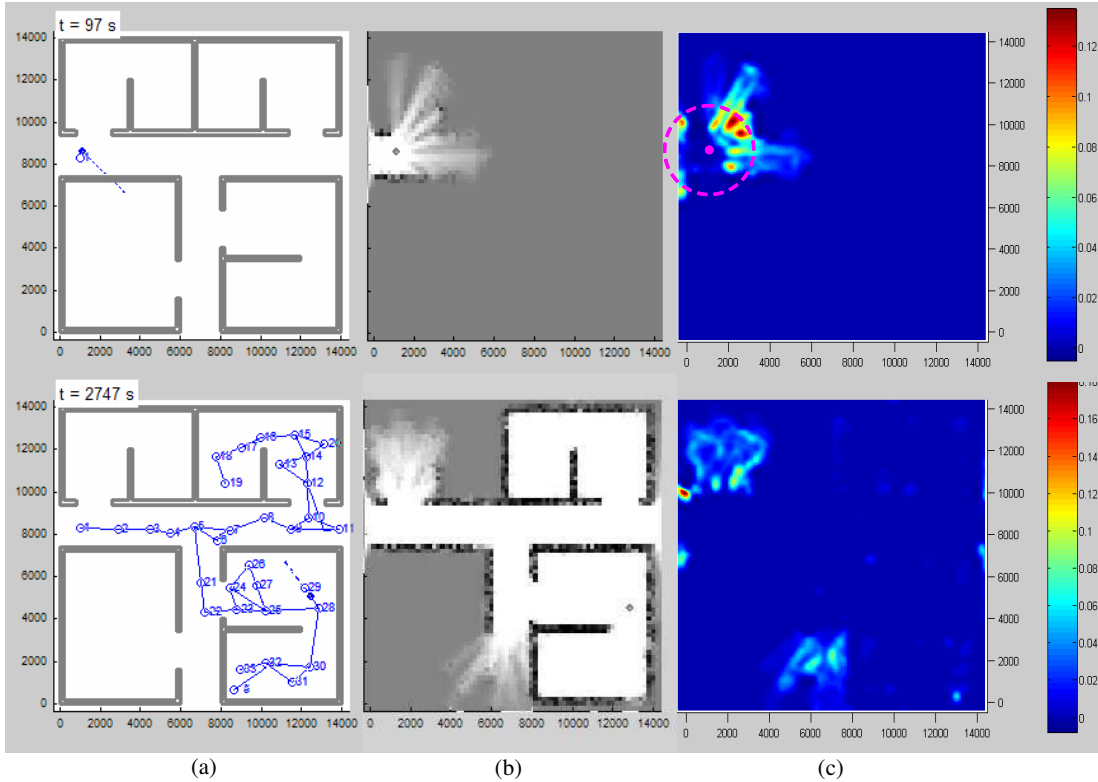
Fig. 1.   Hill climbing exploration, a local search algorithm that selects exploration views with high entropy gradient in the vicinity of the mobile robot. Each row depicts the following information at two different instant times: trajectory and topological map built by the mobile robot while exploring a typical office environment with several rooms and corridors (left); occupancy grid map of the environment (middle); entropy gradient field (right). The top-right graph shows the most frequent exploration scenario wherein the mobile robot seeks and selects line-of-sight, neighbor regions (pink circle) having high entropy gradient. The bottom-right graph depicts an example wherein the maximum entropy gradient value in the robot's vicinity (pink circle) is quite residual. In the latter case, a method is required to recover hill climbing exploration on farther regions with significant gradient (see the arrow).

a topological map which defines a roadmap for path planning and navigation within already explored regions. Stachniss *et al.* used a similar representation to actively closing loops in the context of SLAM [11]. See an example of a topological map overlayed on the bottom-left of Fig. 1.

The topological map is a Voronoi graph $\mathcal{G}$ whose nodes $n \in \mathcal{G}$ represent distinct places in the environment, where the robot has already acquired range measurements, and edges (n_i, n_j), n_i, $n_j \in \mathcal{G}$, represent routes to other places that a robot can traverse safely because are clear of obstacles. For each node, it is stored the cartesian position of the place that it represents. Edges are always assumed herein to be bi-directional, *i.e.* an edge (n_i, n_j) means that mobile robot can either navigate safely from n_i to n_j or from n_j to n_i.

Whenever the robot acquires sensor data, it localizes itself in the topological map, *i.e.* it determines $n_t \in \mathcal{G}$ (see Fig. 2-d). If there is no any place (node) n' in line-of-sight (see Fig. 2-b), or the distance to the closer known place n' is greater than a given distance threshold d_{th} (see Fig. 2-a,c), a new node n^n is created at current robot's position x_t, which becomes the current robot's topological position n_t, and an edge (n_{t-1}, n_t) is created to the node where the robot was previously located in the previous exploration cycle. A straight edge (n_{t-1}, n') is also created if n' exists and

$n' \neq n_{t-1}$ (see Fig. 2-c). The numbering sequence of nodes in the topological map shown in the bottom-left of Fig. 1 represents therefore the sequence of places that the robot has explored until current instant time. A ray casting technique is used in the occupancy grid m to determine if a node $n \in \mathcal{G}$ is visible from a position $\mathbf{x}$; this is denoted as the boolean function $is\_visible(n, \mathbf{x})$. This function is used by the function $nearest\_visible\_place(\mathbf{x})$ to return the nearest visible node from $\mathbf{x}$ (if any) and its distance.

The detailed algorithm for updating the topological map is presented in pseudo-code in Table I. Fig. 2 depicts the four more frequent cases implicit in the algorithm: lines 2 to 11 encompass the cases wherein there is no any visible node at a distance less than d_{th} (see Fig. 2-a-c) and lines 12 to 21 the opposite case (see Fig. 2-d).

B. Topological recovery

The topological map built by the mobile robot during the mapping and exploration process can be used by the hill climbing algorithm to recover from a local minima. If equation (5) does not return any frontier cell, shortest paths from current node n_t to any other node $n_i \neq n_t$, $n_i, n_t \in \mathcal{G}$, are computed using the Dijkstra algorithm [12]. Then the local search algorithm is used to compute frontier cells in the
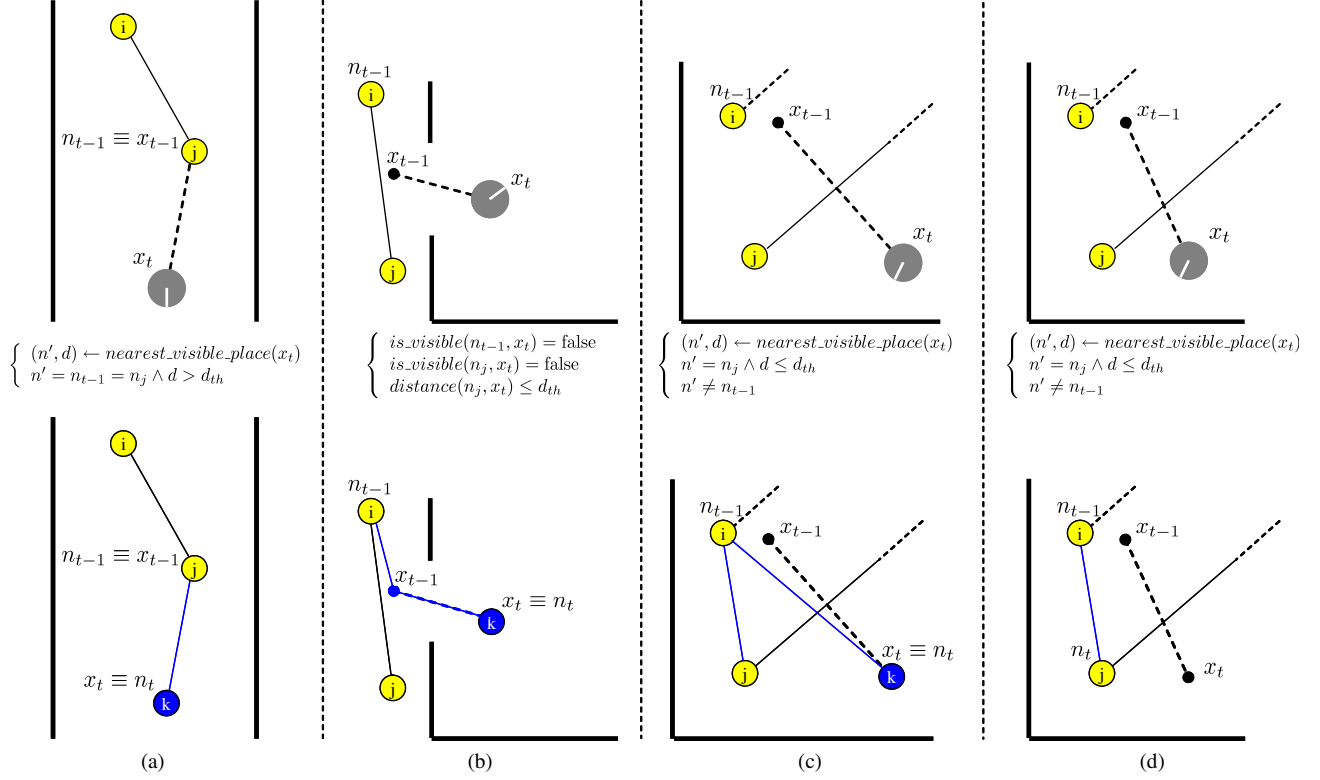
Fig. 2. Typical cases on topological map updates: (a) distance threshold (d_{th}) is exceeded – a new node is created, which becomes the new robot's topological position n_t; no visible place at a distance less than d_{th} – a new place is created, which becomes n_t, and edge (n_{t-1}, n_t) is created with a way point at x_{t-1}; (c) distance d_{th} is exceeded and nodes n_{t-1} and n' are both visible – a new node is created at x_t, which becomes n_t, and edges (n_{t-1}, n_t) and (n_{t-1}, n') are created; (d) nearest visible place n' at a distance less than d_{th}, with $n' \neq n_{t-1}$ – n' becomes n_t and edge (n_{t-1}, n_t) is created.

TABLE I

ALGORITHM FOR UPDATING THE TOPOLOGICAL MAP $\mathcal{G}$.

$update\_topological\_map(x_t, \mathcal{G})$.

```
1:   (n', d) ← nearest_visible_place(x_t)
2:   if (n' does not exist or d > d_th) then
3:       Add a new place n^n at x_t and make n_t ← n_n.
4:       if [(n' does exist and n' = n_{t-1}) or
             is_visible(n_{t-1},x_t)] then
5:           Add a straight edge (n_{t-1}, n_t).
6:       else
7:           Add a polygonal edge (n_{t-1}, n_t) with the way point x_{t-1}.
8:       endif
9:       if (n' does exist and n' ≠ n_{t-1} and
             edge (n', n_{t-1}) does not exist already) then
10:          Add a straight edge (n', n_{t-1}).
11:      endif
12:  else
13:      Make n_t ← n'.
14:      if (n' ≠ n_{t-1} and
             edge (n', n_{t-1}) does not exist already) then
15:          if (is_visible(n_{t-1},x_t)]) then
16:              Add a straight edge (n', n_{t-1}).
17:          else
18:              Add a polygonal edge (n', n_{t-1}) with the way point x_t.
19:          endif
20:      endif
21:  endif
```

vicinity of the position x_i of nodes n_i, *i.e.* for every places visited by the robot since the beginning of the mission, in ascending order of the distance to n_t. As soon as the local search algorithm succeeds for one of the explored nodes n_i^s, the best frontier cell is selected through equation (6), with $x_t = x_i^s$, and the exploration is recovered. Then a shortest path is extracted to navigate the robot from x_t to x_s, where the mapping process is continued. Note that at least one node n_i^s exists for which $\mathcal{N}(x_i^s, \varepsilon) \neq \emptyset$, except if the environment has already been completely explored.

Fig. 3 shows four snapshots of the exploration of a typical office environment by a mobile robot using the proposed method. Each row corresponds to a situation wherein a local minima occurred. On the left, it is shown the topological map (blue), the selected path to recover exploration (red), and the far selected exploration view (green). On the right, it is shown the occupancy grid map immediately before the local minima occurrence (local search radius in pink). The bottom-right occupancy grid map demonstrates that the method was able to explore completely the environment.

IV. COMPLETE, COORDINATED LOCAL SEARCH EXPLORATION WITH MULTIPLE ROBOTS

This section describes how the hill climbing exploration method with topological recovery may be extended to a set
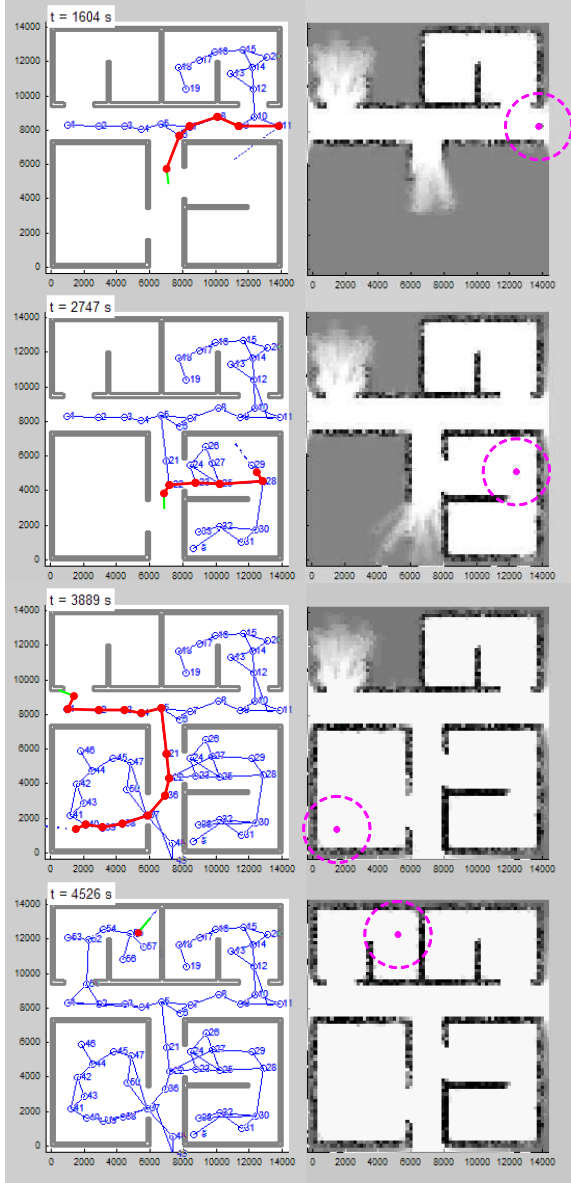
Fig. 3. Complete hill climbing exploration of a typical office environment by a single mobile robot equipped with a ring of 16 sonars.

of cooperative mobile robots. The goal is to take advantage of robots' spatial distribution so as to reduce the time required to complete the exploration. This is viable unless (a) robots share efficiently information and (b) they coordinated properly their exploration actions.

A. Efficiently sharing useful information

In [6], a distributed cooperative architecture model was formulated in the context of the mapping framework described in section II, whereby each robot is altruistically committed to share useful range measurements with other robots. The concept of *useful information* was formally defined using information theory. In summary, each robot

is able to build a map upon measurements from its own range sensors and, whenever it acquires a new batch of measurements, communicates to other robots a subset of useful measurements, using a formal measure of information utility.

B. Coordinated exploration

In spite of the problem regarding local minima, which is only addressed herein, a coordinated exploration method was devised within the same distributed model [7]. The multi-robot version of the exploration algorithm assumes that each robot is aware of the other robot's exploration state: it knows the selected exploration view of each robot and an estimate of its visibility range. This is easily achieved if each robot communicates to its teammates this data, whenever it is changed. Therefore, in the multi-robot case, each robot uses an extension of equation (6), which takes into account the other robots' state. Given a set of frontier cells $\mathcal{N}(x_t, \varepsilon)$ in the robot's vicinity, the best frontier cell is selected using the following criteria:

$$m_i^s = \underset{m_i \in \mathcal{N}(x_t, \varepsilon)}{\operatorname{argmax}} \left[\rho(x_t, m_i).\lambda(m_i).\eta(m_i).\left\| \overrightarrow{\nabla} H(m_i) \right\| \right]. \tag{7}$$

Function $\lambda : m \rightarrow [0, 1]$ computes the non-redundancy coefficient, which is maximum when the robot's sensor field does not overlap with the sensor field of other robots. This ensures that robots tend to explore different regions so as to take maximum advantage from spatial distribution. Function $\eta : m \rightarrow [0, 1]$ computes the non-interference coefficient, which is maximum when there is no any teammate within the robot's sensor field. This situation yields undesirable occlusions and increases the exploration time.

C. Extension of topological recovery to the multi-robot case

The topological recovery presented in section III may be readily extended to the multi-robot case, if three aspects are taken into account.

Firstly, whenever a robot receives measurements from other robot, the algorithm described in Table I is used to include the exploration status of other robots in the topological map, but a robot's identifier must be stored on each created node. This information can be used to verify if some edge exists between a node created by the robot itself and a node created by other robots; otherwise, nodes created by other robots will not be worth to recover exploration.

Secondly, whenever each robot receives measurements from other robot, it checks if there is some connectivity with nodes created by that robot; if not, it tries to find a node created by itself that is visible from the newly created node, in an attempt to ensure connectivity. Sooner of later, this will happen: it suffices that both robots become in line-of-sight at least once.

Thirdly, when a local minima occurs, the nodes where other robots are located should not be included in the topological search described in section III-B, in order to ensure that any path selected by the robot to navigate towards a far exploration view will be not blocked by other robots.
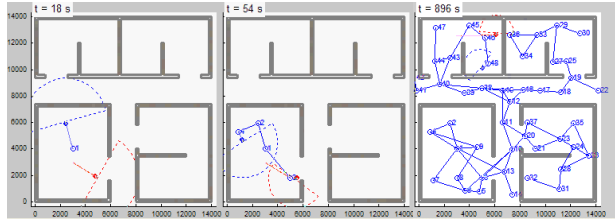
Fig. 4. Exploration of a typical office environment by two cooperative mobile robots: topological map at the beginning of the mission (left), after the two robots becoming in line-of-sight for the first time (middle) and at the end of the mission (right).

Fig. 5. Occupancy grid maps of a typical office environment, obtained by two cooperative mobile robots equipped with a laser range finder: snapshots at three different instant times, being the last one at the end of the mission.

Fig. 4 depicts topological maps built by two cooperative mobile robots (with respect to blue robot) at three different instant times of the mapping and exploration process. At the beginning of the mission (left), node 3, which was created by the other robot (red robot), is still not connected with nodes 1 and 2 created by blue robot. On the middle, nodes 1 and 3 became connected when blue robot received measurements taken by red robot at node 3. On the right, it is shown the topological map at the end of the mission.

Fig. 5 depicts the occupancy grid map of blue robot at three instant times of the mission[2]. Both robots started the mission at the bottom-left room (left). Afterwards blue robot explored the upper-right rooms and red robot the bottom-right rooms. Finally, the mobile robots finished the mission with the exploration of the upper-left rooms.

Given the environment represented in Fig. 4, the performance of teams with varying number of robots is compared in Fig. 6. The graph on the left compares the mission execution time and the graph on the right the speedup measure. This measure reveals how much more efficient are several robots than just one. Teams with up to 4 robots present super-linear performance (speedup greater than one), therefore are proportionally more efficient than a single robot. Being n the teamsize, this means that their mission execution time is lower than the time required by a single robot divided by n. Note the remarkable mission time decrease achieved for $n = 2$.

V. CONCLUSION

Although SLAM has known recently important advances, the action selection problem has received much less attention. This article addressed this problem in the context of

---

[2]Note that these maps are a result of either measurements acquired by the robot itself and measurements received from the other robot.
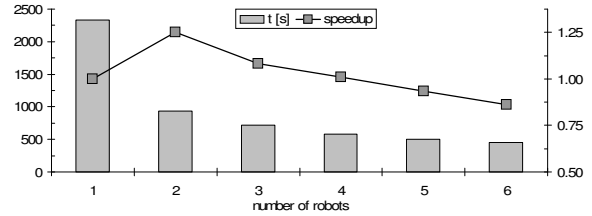

Fig. 6. Performance of multi-robot exploration as a function of teamsize: mission execution time (left) and speedup measure (right).

building a map of an unknown environment, with either a single autonomous mobile robot or multiple cooperative robots, using a local search exploration algorithm. The main contribution was a topological method to ensure that, despite the greedy nature of the algorithm, the environment is completely explored.

The proposed exploration method is going to be used as a module for supporting a robotic mission wherein a set of cooperative mobile robots are deployed in an unknown environment with the goal of exploring it, in a preliminary phase, and then carrying out surveillance missions.

REFERENCES

[1] W. Burgard, M. Moors, C. Stachniss, and F. Schneider. Coordinated multi-robot exploration. *IEEE Transactions on Robotics*, 21(3):376–386, Jun. 2005.

[2] S. Thrun, D. Hähnel, D. Ferguson, M. Montermelo, R. Riebwel, W. Burgard, C. Baker, Z. Omohundro, S. Thayer, and W. Whittaker. A system for volumetric robotic mapping of abandoned mines. In *Proc. of IEEE Int. Conf. on Robotics and Automation (ICRA'2003)*, 2003.

[3] S. Thrun, W. Burgard, and D. Fox. *Probabilistic Robotics*. MIT Press, 2005. ISBN 0-262-20162-3.

[4] B. Yamauchi. Frontier-based exploration using multiple robots. In *Proc. of 2nd Int. Conf. on Autonomous Agents*, pages 47–53, 1998.

[5] R. Rocha, J. Dias, and A. Carvalho. Exploring information theory for vision-based volumetric mapping. In *Proc. of IEEE/RSJ Int. Conf. on Intelligent Robots and Systems (IROS'2005)*, pages 2409–2414, Edmonton, Canada, Aug. 2005.

[6] R. Rocha, J. Dias, and A. Carvalho. Cooperative multi-robot systems: a study of vision-based 3-D mapping using information theory. *Robotics and Autonomous Systems*, 53(3-4):282–311, Dec. 2005.

[7] R. Rocha. Efficient information sharing and coordination in cooperative multi-robot systems. In *Proc. of II European-Latin-American Workshop on Engineering Systems (SELASI'2006)*, Porto, Portugal, Jun. 2006.

[8] H. González-Baños and J.-C. Latombe. Navigation strategies for exploring indoor environments. *Int. Journal of Robotics Research*, 21(10-11):829–848, 2002.

[9] L. Kavraki, P. Svestka, J.-C. Latombe, and M. Overmars. Probabilistic roadmaps for path planning in high-dimensional configuration spaces. *IEEE Tr. on Robotics and Automation*, 12(4):566–580, 1996.

[10] S. LaValle and J. Kuffner. Randomized kinodynamic planning. *Int. Journal of Robotics Research*, 20(5):378–400, May 2001.

[11] C. Stachniss, D. Hähnel, W. Burgard, and G. Grisetti. On actively closing loops in grid-based FastSLAM. *Advanced Robotics*, 19(10):1059–1080, 2005.

[12] T. Cormen, C. Leiserson, R. Rivest, and C. Stein. *Introduction to Algorithms*. MIT Press and McGraw-Hill, second edition, 2001.

[13] IRPS – Intelligent Robotic Porter System project, Jun. 2008. [Online]. Available: http://www.irps-project.net/.