

# Adaptive Tracking Control for Quadrotor Unmanned Flying Vehicle

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**Abstract**—In this work, we introduce adaptive tracking system for quadrotor flying vehicle in the presence of uncertainty. The uncertainty is assumed to be associated with the vehicles payload mass, inertia matrix, actuator faults, and aerodynamic damping force-moment effects and flying environment. The control input combines desired acceleration and proportional-derivative(PD) like auxiliary signals with an adaptation law to learn and compensate uncertainty. Lyapunov method is employed to analyze the closed loop stability of the translational and rotational dynamics of the vehicle. This analysis shows that the tracking error of the translational and rotational dynamics are bounded to zero. Simulation results on a quadrotor vehicle are presented to demonstrate the effectiveness of theoretical arguments of this paper.

## I. INTRODUCTION

In recent years, the use of small scale quadrotor unmanned aerial vehicle (UAV) is growing in military and civilian applications, such as, surveillance, inspection, search and rescue mission in dangerous or awkward environments that are inaccessible for human intervention. In practice, accurate autonomous flight tracking system design for such small scale quadrotor vehicle capable of flying in uncertain environment against uncertainty is challenging because of its underactuated property, coupling between translational and rotational dynamics and nonlinearity associated with the kinematic and dynamical model. Various automatic control methods for quadrotor UAV have been proposed in the literature to achieve desired control stability and tracking performance. These existing results can be classified into two categories as model based linear control and partial model based nonlinear control methods. Classical proportional-integral-derivative (PID) and linear quadratic regulator type controllers were proposed in [2, 3, 4]. In [5, 6, 7], authors used backstepping control technique to address the problem of nonlinear coupling between translational and rotational kinematic and dynamical model of the vehicle. To reduce the steady state tracking errors, authors in [8, 9, 10] included an additional integral action term with the backstepping algorithm [5, 6, 7]. Recently, model based dynamic inversion mechanism have proposed in [11] to achieve hovering flight of the quadrotor UAV system. The existing linear algorithms were designed only for achieving simple hovering flight control strategy by assuming that the model of the vehicle is known *a priori*. As a result, these linear designs may not

be able to achieve desired control stability and tracking performance in the presence of modeling errors and disturbance uncertainties.

Most recent research efforts have been focused on adaptive design to deal with the modeling error uncertainty. Sliding mode technique was applied in [12] to guarantee the stability of the attitude dynamics of the quadrotor UAV vehicle. In [13], authors designed nonlinear control law to guarantee the tracking errors of the linear position and yaw angle of the quadrotor UAV system against *a priori* known disturbance uncertainty. Adaptive backstepping technique was also applied for quadrotor UAV system against modeling error uncertainty in [14, 15].

In this paper, we develop adaptive flight control strategy for quadrotor UAV system to deal with modeling errors and disturbance uncertainty. The proposed design allows the vehicle to fly in uncertain flying environment against the variation of the mass, inertia, actuator faults and nonlinear aerodynamic damping force-moment effects. The altitude, virtual position and attitude controller combines damping and PD like linear control terms with adaptive and robust control terms. Adaptive controller learns and compensates parametric uncertainty associated with the vehicle's model while robust adaptive control deals with the bounded external disturbance uncertainty. The control algorithm and convergence analysis is designed by using Lyapunov second method. It shows that the tracking errors of the altitude, position and orientation are bounded and their bounds converge to zero asymptotically in the Lyapunov sense. To demonstrate these theoretical arguments, simulation example on a quadrotor vehicle is presented.

This paper is organized as follows. In section II, kinematics and dynamics of the four rotor flying vehicles are briefly presented. Algorithms are developed in section III. Various evaluation results are presented in section IV. Finally, conclusion is given in section V.

## II. MODEL DYNAMICS

Let us first model quadrotor flying robot vehicle [1, 2]. To do that, we define the linear position and velocity of the vehicle as  $x_s = [x \ y \ z]^T$  and  $v_s = [v_{s1} \ v_{s2} \ v_{s3}]^T$ . The rotation and angular velocity of the vehicle is chosen as  $\eta = [\phi \ \theta \ \varphi]^T$  and  $\Omega_s = [\Omega_{s1} \ \Omega_{s2} \ \Omega_{s3}]$ . The transformation from body frame to inertial frame is performed by using the following rotation matrix  $R_s \in \mathbb{R}^{3 \times 3}$

$$R_s = \begin{bmatrix} C_\phi C_\varphi & S_\phi S_\theta C_\varphi - C_\phi S_\varphi & C_\phi S_\theta C_\varphi + S_\phi S_\varphi \\ C_\theta S_\varphi & S_\phi S_\theta S_\varphi + C_\phi C_\varphi & C_\phi S_\theta S_\varphi - S_\phi C_\varphi \\ -S_\phi & S_\phi C_\theta & C_\phi C_\theta \end{bmatrix} \quad (1)$$

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where  $C_\theta$  and  $S_\theta$  denotes  $\cos\theta$  and  $\sin\theta$ . The dynamic model for quadrotor vehicle subjected to translational forces and control torques can be written by the following equation

$$\dot{x}_s = v_s \quad (2)$$

$$m_s \dot{v}_s = -m_s g \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + F_t + F_d \quad (3)$$

$$\dot{R}_s = R_s S(\Omega_s) \quad (4)$$

$$I \dot{\Omega}_s = -(\Omega_s \times I \Omega_s) + u_t + u_g + u_a \quad (5)$$

where  $m$  is positive constant,  $I \in \mathbb{R}^{3 \times 3} = \text{diag}[I_x, I_y, I_z]$  is the inertia matrix and  $S(\Omega_s)$  is the skew-symmetric matrix  $S(\Omega_s)$  as defined as follows

$$S(\Omega_s) = \begin{bmatrix} 0 & -\Omega_{s3} & \Omega_{s2} \\ \Omega_{s3} & 0 & -\Omega_{s1} \\ -\Omega_{s2} & \Omega_{s1} & 0 \end{bmatrix} \quad (6)$$

The quadrotor model (2) to (5) can be further simplified as follows [5, 6, 7, 16]

$$\begin{aligned} \ddot{x} &= \frac{1}{p_a} (\cos \phi \sin \theta \cos \varphi + \sin \varphi \sin \phi) u_1 - \\ &\quad Y_{v_{s1}}(v_{s1}) \theta_{vs1} + u_{dx} \\ \ddot{y} &= \frac{1}{p_a} (\cos \phi \sin \theta \sin \varphi - \sin \phi \cos \varphi) u_1 - \\ &\quad Y_{v_{s2}}(v_{s1}) \theta_{vs2} + u_{dy} \\ \ddot{z} &= \frac{1}{p_a} (\cos \phi \cos \theta) u_1 - g - Y_{v_{s3}}(v_{s3}) \theta_{vs3} + u_{dz} \\ \ddot{\phi} &= p_b (\dot{\theta} \dot{\varphi}) + p_{b1} u_2 - p_{b2} f(u) \dot{\theta} - Y_{\Omega_{s1}}(\Omega_{s1}) \theta_{\Omega_{s1}} \\ &\quad + u_{rx} \\ \ddot{\theta} &= p_c (\dot{\phi} \dot{\varphi}) + p_{c1} u_3 + p_{c2} f(u) \dot{\phi} - Y_{\Omega_{s2}}(\Omega_{s2}) \theta_{\Omega_{s2}} \\ &\quad + u_{ry} \\ \ddot{\varphi} &= p_d (\dot{\theta} \dot{\phi}) + p_{d1} u_4 - Y_{\Omega_{s3}}(\Omega_{s3}) \theta_{\Omega_{s3}} + u_{rz} \end{aligned} \quad (7)$$

with  $p_a = m$ ,  $p_b = \left(\frac{p_1}{I_x}\right)$ ,  $p_c = \left(\frac{p_2}{I_y}\right)$ ,  $p_d = \left(\frac{p_3}{I_z}\right)$ ,  $p_1 = (I_y - I_z)$ ,  $p_2 = (I_z - I_x)$ ,  $p_3 = (I_x - I_y)$ ,  $p_{b1} = \left(\frac{I}{I_x}\right)$ ,  $p_{c1} = \left(\frac{I}{I_y}\right)$ ,  $p_{d1} = \left(\frac{I}{I_z}\right)$ ,  $p_{b2} = \left(\frac{I_r}{I_x}\right)$ ,  $p_{c2} = \left(\frac{I_r}{I_y}\right)$ ,  $Y_{v_{s1}}(v_{s1}) \theta_{vs1} = \frac{\delta_x}{m}$ ,  $Y_{\Omega_{s1}}(\Omega_{s1}) \theta_{\Omega_{s1}} = \frac{\pi_{1x}}{I_x}$ ,  $Y_{v_{s2}}(v_{s2}) \theta_{vs2} = \frac{\delta_y}{m}$ ,  $Y_{\Omega_{s2}}(\Omega_{s2}) \theta_{\Omega_{s2}} = \frac{\pi_{1y}}{I_y}$ ,  $Y_{v_{s3}}(v_{s3}) \theta_{vs3} = \frac{\delta_z}{m}$ ,  $Y_{\Omega_{s3}}(\Omega_{s3}) \theta_{\Omega_{s3}} = \frac{\pi_{1z}}{I_z}$ ,  $Y_{v_{s1}}(v_{s1}) = [v_{s1}, v_{s1}|v_{s1}]$ ,  $Y_{v_{s2}}(v_{s1}, v_{s2}) = [v_{s2}, v_{s2}|v_{s1}]$ ,  $Y_{v_{s3}}(v_{s3}) = [v_{s3}, v_{s3}|v_{s3}]$ ,  $Y_{\Omega_{s1}}(\Omega_{s1}) = [\Omega_{s1}, \Omega_{s1}|\Omega_{s1}]$ ,  $Y_{\Omega_{s2}}(\Omega_{s2}) = [\Omega_{s2}, \Omega_{s2}|\Omega_{s2}]$ ,  $Y_{\Omega_{s3}}(\Omega_{s3}) = [\Omega_{s3}, \Omega_{s3}|\Omega_{s3}]$ ,  $\theta_{vs1} = \left[\frac{\xi_1}{m}, \frac{\xi_2}{m}\right]^T$ ,  $\theta_{vs2} = \left[\frac{\xi_3}{m}, \frac{\xi_4}{m}\right]^T$ ,  $\theta_{vs3} = \left[\frac{\xi_5}{m}, \frac{\xi_6}{m}\right]^T$ ,  $\theta_{\Omega_{s1}} = \left[\frac{\xi_1}{I_x}, \frac{\xi_2}{I_x}\right]^T$ ,  $\theta_{\Omega_{s2}} = \left[\frac{\xi_3}{I_y}, \frac{\xi_4}{I_y}\right]^T$ ,  $\theta_{\Omega_{s3}} = \left[\frac{\xi_5}{I_z}, \frac{\xi_6}{I_z}\right]^T$ ,  $f(u) = [\omega_1 - \omega_2 + \omega_3 - \omega_4]$ ,  $u_d(t) = [u_{dx}(t), u_{dy}(t), u_{dz}(t)]^T$  and  $u_r(t) = [u_{rx}(t), u_{ry}(t), u_{rz}(t)]^T$ .

### III. CONTROL DESIGN AND STABILITY ANALYSIS

It is very challenging task to design autonomous flight control system for a small size quadrotor flying vehicle to ensure control stability and tracking against modeling

error and disturbance uncertainties in uncertain flying environment. This is mainly because of unpredictable variation in flying environment, very strong dynamic coupling and nonlinearities associated with the translational and rotational dynamics of the vehicle. In view of the model (7), we can notice that the mass, inertia, aerodynamic damping forces and torques are uncertain as they may vary with uncertain flight mission and flying environment. To deal with this problem, we introduce adaptive tracking algorithms to learn and cope with uncertain modeling error and disturbance uncertainties entering into the system. The proposed design and stability analysis are assumed to be satisfied the following conditions: A1: The linear and angular position and their first and second derivatives are bounded. A2: The linear and angular position are available for measurement. A3: The external disturbances and their time derivatives are assumed to be bounded.

#### A. Design and Convergence Analysis

Let us first introduce the following altitude tracking control algorithm [16]

$$\begin{aligned} u_1 &= \frac{p_a}{\cos \phi \cos \theta} (\ddot{z}_d + g + Y_{v_{s3}}(v_{s3}) \hat{\theta}_{vs3} + u_{bz} + \\ &\quad \alpha_{z1} \dot{e}_z + k_z S_z) \\ u_{bz} &= -\hat{b}_z \text{sign}(S_z) \\ \dot{\hat{\theta}}_{vs3} &= -\gamma_z Y_{v_{s3}}^T(v_{s3}) S_z \\ \dot{\hat{b}}_z &= \gamma_{pa} \text{sign}^T(S_z) S_z \end{aligned} \quad (8)$$

with  $e_z = (z_d - z)$ ,  $\dot{e}_z = (\dot{z}_d - \dot{z})$ ,  $\alpha_z \succ 0$ ,  $\tilde{\theta}_{vs3} = (\hat{\theta}_{vs3} - \theta_{vs3})$ ,  $\tilde{b}_z = (b_z - \hat{b}_z)$ ,  $\gamma_{pa} \succ 0$ ,  $k_z \succ 0$ ,  $\alpha_{z1} > \alpha_z$ ,  $\gamma_z = \text{diag}[\gamma_{z1}, \gamma_{z2}]$ ,  $\gamma_{z1} > 0$  and  $\gamma_{z2} > 0$ . We now develop virtual position controller for generating desired rolling and pitching motion to keep the flying vehicle over the desired point. The desired rolling motion in x axis is generated by the following input

$$\begin{aligned} p_{xi} &= \frac{p_a}{u_1} (\ddot{x}_d + Y_{v_{s1}}(v_{s1}) \hat{\theta}_{vs1} + \alpha_{x1} \dot{e}_x + u_{bx} + k_x M_x) \\ \dot{\hat{\theta}}_{vs1} &= -\gamma_x Y_{v_{s1}}^T(v_{s1}) M_x, u_{bx} = -\hat{b}_x \text{sign}(M_x) \\ \dot{\hat{b}}_x &= \gamma_{dx} \text{sign}^T(M_x) M_x \end{aligned} \quad (9)$$

with  $M_x = (\dot{e}_x + \alpha_x e_x)$ ,  $\alpha_x \succ 0$ ,  $\gamma_{dx} \succ 0$ ,  $\tilde{\theta}_{vs1} = (\hat{\theta}_{vs1} - \theta_{vs1})$ ,  $\tilde{b}_x = (b_x - \hat{b}_x)$ ,  $k_x \succ 0$ ,  $\alpha_{x1} > \alpha_x$ ,  $\gamma_x = \text{diag}[\gamma_{x1}, \gamma_{x2}]$ ,  $\gamma_{x1} > 0$  and  $\gamma_{x2} > 0$ . The pitching motion in y direction can be obtained by using the following virtual input

$$\begin{aligned} p_{yi} &= \frac{p_a}{u_1} (\ddot{y}_d - Y_{v_{s2}}(v_{s2}) \hat{\theta}_{vs2} + \alpha_{y1} \dot{e}_y + u_{by} + k_y M_y) \\ \dot{\hat{\theta}}_{vs2} &= -\gamma_y Y_{v_{s2}}^T(v_{s2}) M_y, u_{by} = -\hat{b}_y \text{sign}(M_y) \\ \dot{\hat{b}}_y &= \gamma_{dy} \text{sign}^T(M_y) M_y \end{aligned} \quad (10)$$

with  $M_y = (\dot{e}_y + \alpha_y e_y)$ ,  $\alpha_y \succ 0$ ,  $e_x = (x_d - x)$ ,  $e_y = (y_d - y)$ ,  $\gamma_{dy} \succ 0$ ,  $\tilde{\theta}_{vs2} = (\hat{\theta}_{vs2} - \theta_{vs2})$ ,  $\tilde{b}_y = (b_y - \hat{b}_y)$ ,  $k_y \succ 0$ ,  $\alpha_{y1} > \alpha_y$  and  $\gamma_y = \text{diag}[\gamma_{y1}, \gamma_{y2}]$ ,  $\gamma_{y1} > 0$  and  $\gamma_{y2} > 0$ . We now design algorithms for stabilizing and tracking the attitude dynamics of the vehicle against

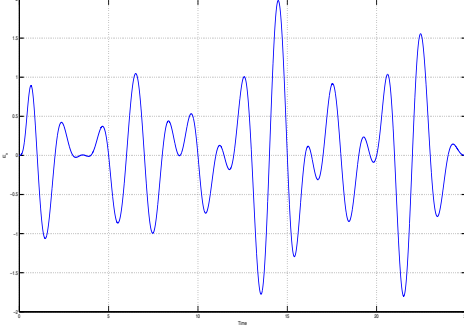


Fig. 1. State dependant disturbance uncertainty acting along with  $x$  axis in newton.

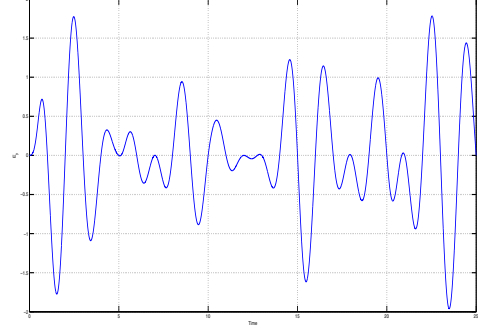


Fig. 2. State dependant disturbance uncertainty included in  $y$  axis in newton.

modeling error and disturbance uncertainties. We propose the following input for generating the desired rolling moment

$$\begin{aligned} u_2 &= \frac{1}{p_b} [\ddot{\phi}_d - \hat{M}_1 \mathcal{K}_l(\Omega_{s2}, \Omega_{s3}) + \hat{M}_2 \mathcal{M}_l(u, \Omega_{s2}) + \\ &\quad Y_{\Omega_{s1}}(\Omega_{s1}) \hat{\theta}_{\Omega_{s1}} + u_{r\phi} + \alpha_{\phi_1} \dot{e}_\phi + k_\phi N_\phi] \\ u_{r\phi} &= -\hat{c}_\phi \text{sign}(N_\phi), \quad \dot{\hat{\theta}}_{\Omega_{s1}} = -\gamma_\phi Y_{\Omega_{s1}}^T(\Omega_{s1}) N_\phi, \\ \dot{\hat{c}}_\phi &= \gamma_{pb1} \text{sign}^T(N_\phi) N_\phi, \quad \dot{\hat{M}}_1 = \gamma_{M1} \mathcal{K}_l^T(\Omega_{s1}, \Omega_{s2}) N_\phi \\ \dot{\hat{M}}_2 &= -\gamma_{M2} \mathcal{M}_l^T(u, \Omega_{s2}) N_\phi \end{aligned} \quad (11)$$

with  $k_\phi \succ 0$ ,  $\mathcal{K}_l(\Omega_{s2}, \Omega_{s3}) = \dot{\theta}\dot{\phi}$ ,  $\mathcal{M}_l(u, \Omega_{s2}) = f(u)\dot{\theta}$ ,  $N_\phi = (\dot{e}_\phi + \alpha_\phi e_\phi)$ ,  $e_\phi = (\phi_d - \phi)$ ,  $\alpha_\phi \succ 0$ ,  $\alpha_{\phi_1} > \alpha_\phi$ ,  $\gamma_{pb1} \succ 0$ ,  $\gamma_\phi = \text{diag}[\gamma_{\phi_1}, \gamma_{\phi_2}]$ ,  $\gamma_{M2} \succ 0$ ,  $\gamma_{M1} \succ 0$ ,  $\hat{\theta}_{\Omega_{s1}} = (\hat{\theta}_{\Omega_{s1}} - \theta_{\Omega_{s1}})$ ,  $\tilde{c}_\phi = (c_\phi - \hat{c}_\phi)$ ,  $\tilde{M}_2 = (\hat{M}_2 - M_2)$ ,  $\tilde{M}_1 = (M_1 - \hat{M}_1)$  and  $\phi_d$  is the reference rolling angle and calculated from the relationship  $\phi_d = \arcsin(p_x \sin(\varphi_d) - p_y \cos(\varphi_d))$ . We design the following input to generate the pitching moment

$$\begin{aligned} u_3 &= \frac{1}{p_{c1}} [\ddot{\theta}_d - \hat{M}_3 \mathcal{B}_l(\Omega_{s1}, \Omega_{s3}) - \hat{M}_4 \mathcal{C}_l(u, \Omega_{s1}) + \\ &\quad Y_{\Omega_{s2}}(\Omega_{s2}) \hat{\theta}_{\Omega_{s2}} + u_{r\theta} + \alpha_{\theta_1} \dot{e}_\theta + k_\theta L_\theta] \\ u_{r\theta} &= -\hat{c}_\theta \text{sign}(L_\theta), \quad \dot{\hat{\theta}}_{\Omega_{s2}} = -\gamma_\theta Y_{\Omega_{s2}}^T(\Omega_{s2}) L_\theta, \\ \dot{\hat{c}}_\theta &= \gamma_{pc1} \text{sign}^T(L_\theta) L_\theta, \quad \dot{\hat{M}}_4 = \gamma_{M4} \mathcal{C}_l^T(u, \Omega_{s1}) L_\theta, \\ \dot{\hat{M}}_3 &= \gamma_{M3} \mathcal{B}_l^T(\Omega_{s1}, \Omega_{s3}) L_\theta \end{aligned} \quad (12)$$

with  $\mathcal{C}_l(u, \Omega_{s1}) = f(u)\dot{\phi}$ ,  $\mathcal{B}_l(\Omega_{s1}, \Omega_{s3}) = \dot{\phi}\dot{\phi}$ ,  $L_\theta = (\dot{e}_\theta + \alpha_\theta e_\theta)$ ,  $e_\theta = (\theta_d - \theta)$ ,  $k_\theta \succ 0$ ,  $\theta_d$  is the reference pitching angle,  $\alpha_\theta \succ 0$ ,  $\alpha_{\theta_1} > \alpha_\theta$ ,  $\gamma_{pc1} \succ 0$ ,  $\gamma_\theta \in \mathbb{R}^{2 \times 2}$ ,  $\gamma_{M3} \succ 0$ ,  $\gamma_{M4} \succ 0$ ,  $\hat{\theta}_{\Omega_{s2}} = (\hat{\theta}_{\Omega_{s2}} - \theta_{\Omega_{s2}})$ ,  $\tilde{c}_\theta = (-\hat{c}_\theta + c_\theta)$ ,  $\tilde{M}_3 = (-\hat{M}_3 + M_3)$  and  $\tilde{M}_4 = (-\hat{M}_4 + M_4)$ . Using  $\varphi_d$ ,  $\phi_d$ ,  $p_x$  and  $p_y$ ,  $\theta_d$  can be calculated from the relationship  $\theta_d = \arcsin\left(\frac{p_x \sin(\varphi_d) + p_y \cos(\varphi_d)}{\cos(\phi_d)}\right)$ . Finally, the following input

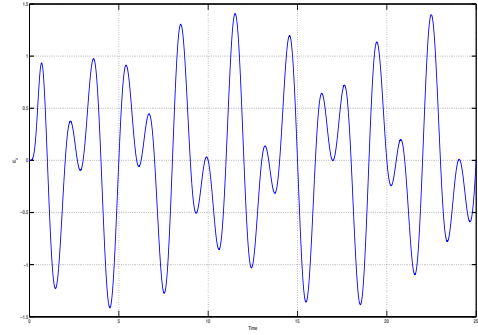


Fig. 3. State dependant disturbance uncertainty included with  $z$  axis in newton.

is designed to develop the desired yaw moment

$$\begin{aligned} u_4 &= \frac{1}{p_{d1}} [\ddot{\psi}_d - \hat{M}_5 \mathcal{D}_l(\Omega_{s1}, \Omega_{s2}) + Y_{\Omega_{s3}}(\Omega_{s3}) \hat{\theta}_{\Omega_{s3}} + \\ &\quad u_{r\psi} + \alpha_{\varphi_1} \dot{e}_\varphi + k_\varphi P_\varphi] \\ u_{r\psi} &= -\hat{c}_\psi \text{sign}(P_\varphi), \quad \dot{\hat{\theta}}_{\Omega_{s3}} = -\gamma_\varphi Y_{\Omega_{s3}}^T(\Omega_{s3}) P_\varphi \\ \dot{\hat{c}}_\psi &= \gamma_{pd1} \text{sign}^T(P_\varphi) P_\varphi, \quad \dot{\hat{M}}_5 = \gamma_M \mathcal{D}_l^T(\Omega_{s1}, \Omega_{s2}) P_\varphi \end{aligned} \quad (13)$$

with  $\mathcal{D}_l(\Omega_{s1}, \Omega_{s2}) = \dot{\theta}\dot{\phi}$ ,  $P_\varphi = (\dot{e}_\varphi + \alpha_\varphi e_\varphi)$ ,  $e_\varphi = (\varphi_d - \varphi)$ ,  $k_\varphi \succ 0$ ,  $\varphi_d$  is the reference yaw angle,  $\alpha_\varphi \succ 0$ ,  $\alpha_{\varphi_1} > \alpha_\varphi$ ,  $\gamma_{pd1} \succ 0$ ,  $\gamma_M \succ 0$ ,  $\gamma_\varphi = \text{diag}[\gamma_{\varphi_1}, \gamma_{\varphi_2}]$ ,  $\hat{\theta}_{\Omega_{s3}} = (\hat{\theta}_{\Omega_{s3}} - \theta_{\Omega_{s3}})$ ,  $\tilde{M}_5 = (M_5 - \hat{M}_5)$  and  $\tilde{c}_\psi = (\hat{c}_\psi - c_\psi)$ . Using Lyapunov analysis, we can show state that all the error states in the closed loop tracking error systems formulated by equations (7), (8) to (13) are bounded and their bounds asymptotically converge to zero [16].

#### IV. EVALUATION RESULTS

In this section, we evaluate the proposed algorithms on a simulated model of quadrotor flying robot vehicle. Extensive simulation studies have been conducted to evaluate the flight control stability and tracking property of the proposed algorithms with respect to varying mass and inertia matrix, aerodynamic damping effect and external disturbances. The

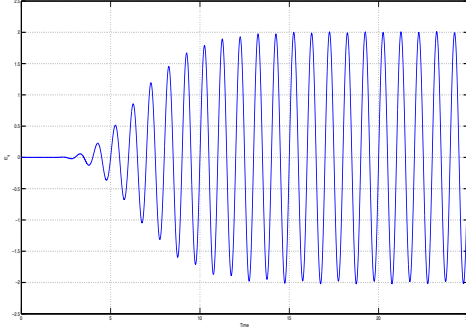


Fig. 4. State dependant disturbance with roll angle axis in newton-meters.

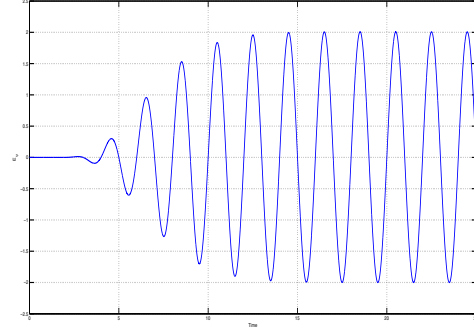


Fig. 6. State dependant disturbance with yaw angle axis in newton-meters.

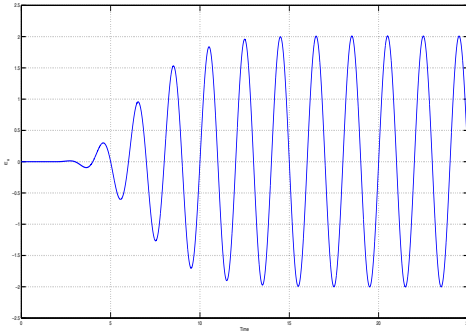


Fig. 5. State dependant disturbance uncertainty with pitch angle axis in newton-meters.

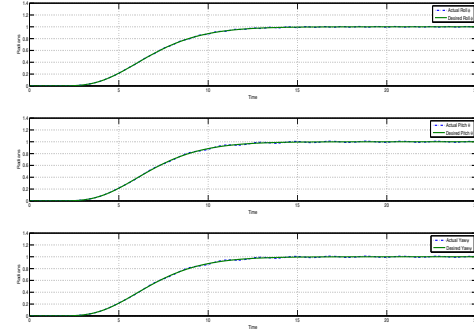


Fig. 7. Desired and actual attitude tracking of roll, pitch and yaw angles.

time varying desired trajectories  $x_d$ ,  $y_d$  and  $z_d$  are chosen as  $x_d(t) = (1 - e^{-5t^3})\sin(10t)$ ,  $y_d(t) = (1 - e^{-5t^3})\cos(10t)$  and  $z_d(t) = (1 - e^{-5t^3})$ . The desired trajectory for  $\psi_d$  is generated by using the output of the transfer function  $H(s) = \frac{4}{s^2 + 4s + 4}$ . The control design parameters are chosen arbitrarily as  $\gamma_{M1} = 0.5$ ,  $\gamma_{M2} = 2$ ,  $\gamma_{M3} = 2$ ,  $\gamma_{M4} = 2$ ,  $\gamma_{M5} = 2$ ,  $\gamma_x = [5 \ 0; 0 \ 3]$ ,  $\gamma_y = [5 \ 0; 0 \ 3]$ ,  $\gamma_z = [0.01 \ 0; 0 \ 0.01]$ ,  $\gamma_{pa} = 2$ ,  $\gamma_{pb1} = 2$ ,  $\gamma_{pc1} = 2$ ,  $\gamma_{pd1} = 2$ ,  $\alpha_x = 10$ ,  $\alpha_y = 10$ ,  $\alpha_z = 70$ ,  $\alpha_\phi = 2$ ,  $\alpha_\theta = 2$ ,  $\alpha_\psi = 2$ ,  $k_x = 1$ ,  $k_y = 1$ ,  $k_z = 1$ ,  $k_\phi = 1$ ,  $k_\theta = 1$  and  $k_\psi = 1$ . It is noted that these parameters can be increased to improve the tracking performance further. The mass, inertia and nonlinear aerodynamic damping parameters of the vehicle are considered relatively large values as  $m = 8 \text{ kg}$ ,  $I_x = 2 \text{ Nm} \cdot \frac{s^2}{rad}$ ,  $I_y = 2.5 \text{ Nm} \cdot \frac{s^2}{rad}$ ,  $I_z = 3 \text{ Nm} \cdot \frac{s^2}{rad}$ ,  $I_r = 0.3 \text{ Nm} \cdot \frac{s^2}{rad}$ ,  $\zeta_1 = 2 \text{ N} \cdot \frac{s}{m}$ ,  $\zeta_2 = 5 \text{ N} \cdot \frac{s}{m}$ ,  $\zeta_3 = 6 \text{ N} \cdot \frac{s}{m}$ ,  $\zeta_4 = 6 \text{ N} \cdot \frac{s}{m}$ ,  $\zeta_5 = 7 \text{ N} \cdot \frac{s}{m}$ ,  $\zeta_6 = 3 \text{ N} \cdot \frac{s}{m}$ ,  $\xi_1 = 1 \text{ N} \cdot \frac{s}{m}$ ,  $\xi_2 = 3 \text{ N} \cdot \frac{s}{m}$ ,  $\xi_3 = 6 \text{ N} \cdot \frac{s}{m}$ ,  $\xi_4 = 6 \text{ N} \cdot \frac{s}{m}$ ,  $\xi_5 = 8 \text{ N} \cdot \frac{s}{m}$  and  $\xi_6 = 3 \text{ N} \cdot \frac{s}{m}$ . In our evaluation, the state dependant disturbance uncertainties along with the translational and rotational dynamics (7) are chosen as  $E_x = y\sin(pit) + z\sin(pit)$ ,  $E_y = z\sin(pit) + x\cos(pit)$ ,  $E_z = y\sin(pit) + x\cos(pit)$ ,  $E_\phi = \theta\sin(2pit) + \psi\sin(2pit)$ ,  $E_\theta = \phi\cos(pit) + \psi\sin(pit)$  and  $E_\varphi = \phi\cos(pit) + \theta\sin(pit)$ . The time history of these disturbances is shown in Figs. 1 to 6. Using with these parameters, we implement the design

on the simulated quadrotor flying vehicle. The evaluation results are depicted in Figs 7 to 8. From these results, we can observe that the position, altitude and attitude of the flying vehicles converges to the given reference position, altitude and attitude even in the presence of large modeling errors and disturbance uncertainties. These evaluation results demonstrate the validity of the control stability and tracking property of the proposed design.

## V. CONCLUSION

In this paper, we have introduced adaptive tracking system for quadrotor flying vehicle to deal with the modeling errors and disturbance uncertainties. Algorithm designs and their stability analysis have been established through Lyapunov-like energy function by assuming that all the states are available for feedback. The design can learn and compensate the bounds of the modeling errors and disturbances while maintain the bounded stability and tracking control property of the whole closed loop system in Lyapunov sense. To illustrate these arguments, simulations results on a quadrotor vehicle have been presented.

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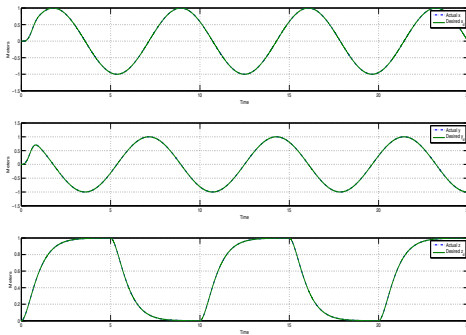


Fig. 8. Desired and actual tracking in x, y and z axis.

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